SME Failures Under Large Liquidity Shocks: An Application to the COVID-19 Crisis [*]

Pierre Olivier GOURINCHAS[†]
International Monetary Fund

Şebnem KALEMLI-ÖZCAN[‡]
University of Maryland

Veronika PENCIAKOVA[§]
Federal Reserve Bank of Atlanta

Nick SANDER[¶]
Bank of Canada

February 2024

Abstract

We study the effects of financial frictions on firm exit when firms face large liquidity shocks. We develop a simple model of firm cost-minimization, where firms' borrowing capacity to smooth temporary shocks to liquidity is limited. In this framework, firm exit arises from the interaction between this financial friction and fluctuations in cash flow due to aggregate and sectoral changes in demand conditions, as well as more traditional shocks to productivity. To evaluate the implications of our model, we use firm level data on small and medium sized enterprises (SMEs) in 11 European countries. We confirm that our framework is consistent with official failure rates in 2017-2019, a period characterized by standard business cycle fluctuations. To capture a large liquidity shock, we apply our framework to the COVID-19 crisis. We find that, absent government support, SME failure rates would have increased by 6.01 percentage points, putting 3.1% of employment at risk. Our results also show that in the presence of financial frictions and in the absence of government support, the firms failing due to COVID have similar productivity and growth to firms that survive COVID.

[*]This paper was circulated before under the title: "COVID-19 and SME Failures." We thank Ryan Decker, Philippe Martin, Brent Neiman, Xavier Ragot, David Sraer, David Thesmar, Jianlan Wang, colleagues at the IMF, seminar participants at the BIS, OECD, Banco Central de Chile, SF FED, the 2019 "Conference on the Financial Consequences of the COVID-19 Pandemic," organized by the Journal of Finance and the Fama-Miller Center (Chicago, Booth), the 2020 Federal Reserve System Macroeconomics Conference, and the ASSA 2021 meetings for useful comments. The views expressed here are those of the authors and do not necessarily reflect the views of the International Monetary Fund, Federal Reserve System, Board of Governors, the Bank of Canada, or their staff.

[†]Also CEPR (London). pog@berkeley.edu

[‡]Also NBER (Cambridge, MA) and CEPR (London). kalemli@umd.edu

[§]email: veronika.penciakova@atl.frb.org.

[¶]email: NSander@bank-banque-canada.ca.

1 Introduction

Firm exit is an important contributor to macroeconomic boom-bust cycles. In the United States, 7.5% of firms exit annually, with both the level and cyclicality of this exit being primarily driven by small firms (Crane et al., 2022). The high rate of firm exit, especially during recessionary periods, raises two key questions: what types of firms exit during downturns; and what types of firms do untargeted government interventions save?

Most existing theoretical models of firm dynamics put firms' productivity and shocks to that productivity at the core of firms' exit decision, which generates higher exit rates from low productivity firms and "cleansing" recessions.[1] Many models therefore ignore both the types of frictions and shocks that contribute to the exit of growing, productive firms and sluggish recoveries. Specifically, small and medium sized enterprises (SMEs) have been shown to be financially constrained both during normal times and crises by a large literature in finance. Moreover, besides shocks to productivity and disruptions in the credit market, firms also face changing demand conditions, especially during recessions.[2] Precisely the interaction between these firm financial frictions and changing market demand conditions could be important in explaining firm exit.

In this paper, we use a simple firm cost-minimization model, combined with firm balance sheet data, to study the impact of firm financial frictions on SME exit, under a variety of shocks that may affect firm liquidity. These shocks arise from changes in aggregate and sectoral demand and supply conditions. For instance, when consumer demand declines, firm cash flow falls. Firms fail, even after shedding workers and/or closing temporarily, when they cannot cover the fall in cash flow due to financial constraints. Firm failure therefore arises from the interaction between negative non-financial shocks to liquidity and firm financial frictions because firms, even when financial markets function normally, cannot fully smooth these shocks by borrowing from the financial sector.[3]

A key feature of our framework, relative to previous modeling exercises, is that it uses pre-crisis firm level *data* to summarize the initial distribution of firm health and profitability, and models how firms *adjust* their production decisions when faced with a series of both demand and supply shocks. With this approach, we can estimate the impact of shocks on individual

[1]See Clementi and Palazzo, 2016, Lee and Mukoyama, 2018, Samaniego, 2008. See also Caballero and Hammour, 1994, Caballero et al., 2008, Foster et al., 2001, Klette and Kortum, 2003, and Samaniego, 2010 regarding the "cleansing" effects of recessions.

[2]See Ates and Saffie, 2021, Choi, 2013, Khan and Thomas, 2013, Kalemli-Ozcan et al., 2022, and Sedláček and Sterk, 2017 regarding sluggish recoveries arising from frictions, including financial constraints. Note that models that incorporate financial frictions often focus on the effect of shocks in the credit market. See Ayres and Raveendranathan, 2021, and Khan and Thomas, 2013.

[3]This type of financial friction has been shown to be empirically relevant for SMEs (e.g. Caglio et al., 2021, Dinlersoz et al., 2019, Gopinath et al., 2017

firms, and consequently the effect of shocks on the distribution of surviving versus failing firms. firms fail. We can also evaluate counterfactual scenarios in which different degrees and forms of government support are implemented to evaluate the distributional, sectoral, and aggregate impact of policy on SME failure, as well as the associated costs and benefits. Consequently, we can answer whether policy saves more productive, high growth firms or less productive, slow growth firms. Furthermore, our framework can be utilized in real time, providing a powerful tool for policymakers to gain potential insights on firm health quickly at the onset of a crisis.

Our starting point is a firm cost minimization model in which firms face a set of liquidity shocks (in the form of sectoral demand and supply shocks) that affect firm cash flow. Total demand for a firm's output in each sector is affected by both aggregate and sector-specific demand shocks. An aggregate demand shock captures changes in aggregate expenditures and affects all firms proportionately. A sector-specific demand shock reflects changes in the pattern of household spending resulting from changes in preferences for certain goods. On the supply side, prices are fixed and output is demand determined. Firms must adjust variable intermediate inputs (labor and materials) to meet demand, subject to labor supply shocks. Meeting demand in this constrained environment may lead to further cash flow deterioration, in which case firms may prefer to temporarily shut down rather than produce (*mothball*).

In the model, firms fail if pre-shock cash balances plus current period cash flow are insufficient to cover the interest payments on pre-existing debt for the year. Two aspects of our failure criterion are worth noting. First, while SMEs face financial constraints in terms of borrowing to smooth out the original shock, our exit criterion recognizes that they have *some* capacity to smooth cash flow in times of temporary stress. We allow firms to hold their existing debt levels constant and require them only to make interest payments on this debt. Moreover, by categorizing firms as failing only if their end-of-year cash balance is negative, we are implicitly assuming firms can obtain credit to remain liquid during temporary cash deficits, provided their remaining profits for the year are sufficient to allow full repayment of this credit. Second, the failure criterion is based on firm *illiquidity* as opposed to insolvency. Empirical evidence shows that SMEs face liquidity constraints that likely dominate solvency concerns during large liquidity shocks.[4] In such instances, promising (i.e. solvent) firms can fail along with weaker (i.e. insolvent) firms.

We use firm balance sheet and income statement data from Bureau van Djik's Orbis. We focus on SMEs in a sample of eleven European countries. In the European Union, SMEs account for 99.8% of all employer firms, 59.4% of private sector employment, and 53.1% of gross output.[5] For each firm, we observe sales, labor and material costs, cash balances, and interest

[4]See Acharya and Steffen (2020).

[5]SME contribution to the economy is derived using Eurostat's Structural Business Statistics for the available

payments, which are used to estimate changes in cash flow. We also observe various metrics of firm health, such as labor productivity, revenue and employment growth, and leverage. Using the model and a sequence of shocks, for each firm we can estimate changes in cash flow and evaluate the failure criterion, as well as fully characterize and compare the labor productivity, growth, and leverage of failing versus surviving firms.

We first consider a "typical" year scenario for 2017-2019—years in which our sample of eleven European countries faced modest economic shocks. We combine country-specific aggregate and 1-digit sectoral shocks, calculated using Eurostat data, with the prior year's firm level Orbis data, and our model to predict firm failures. The difference between our estimated failure rates and actual failure rates at the country-sector-year level is on average only 0.69 percentage points, which is less than 10% of the 8.97% average failure rate over the period. We also compare firms classified as failing versus classified as surviving on simple profitability and liquidity measures. Consistent with the predictions of both the empirical literature and modelling approaches where exit is based on solvency, we find that firms predicted to fail were less productive, grew slower, had less cash on hand, and were more leveraged than those predicted to survive.

We then apply our framework to COVID-19, which was an unprecedented shock to a vast number of firms' cash flows. As such, it is precisely the type of situation where our framework can provide insight on the underlying sources of economic vulnerability and on the potential implications of various policy interventions. To model COVID-19, we assume that shocks hit at the end of February 2020 and the subsequent stringent social distancing period lasts 8 weeks. During these 8 weeks, each sector in the economy is affected by four types of shocks: sector-specific demand shocks, reflecting changes in the pattern of spending away from social consumption; declines in overall spending due to precautionary savings and falls in income; productivity losses from shifting to remote work; and labor restrictions reflecting lockdowns and workplace social distancing. At the end of lockdown, sectoral supply shocks return to their pre-COVID levels, while aggregate demand evolves according to IMF quarterly projections and sector-specific demand reverts back to normal slowly.

To understand sources of vulnerability to the COVID-19 crisis, we first estimate failure rates absent government intervention. Under this baseline scenario, COVID would have raised overall SME failure rates by 6.01 percentage points (relative to a non-COVID 2020 scenario). With excess failure rates above 19 percentage points, the most vulnerable sectors are Arts, Entertainment, & Recreation and Education. We find that most of the sectoral variation in failure rates results from large falls in sector-specific demand. Vulnerability also varies considerably across countries. For instance, the excess failure rate in Romania is estimated at 2.37 percent-

set of sectors. Note that SMEs account for over 50% of output even when all the sectors of the economy are considered, as shown in Kalemli-Ozcan et al. (2019).

age points (pp), 5.27 pp in France, and 10.30 pp in Italy. An important source of vulnerability in a country like Italy is that firms entered COVID with considerably lower cash balances and higher debt burdens than firms in other countries, like France, that faced similar shocks. Italian firms will therefore experience larger cash shortfalls than French firms in response to the same set of shocks. Because our financial friction limits the time firms have to recover cash deficits, Italian firms fail at a higher rate than French ones.

The baseline scenario highlights that many additional firms are at risk of failure due to COVID-19. Using firm level data, we investigate the characteristics of these firms. Specifically, we compare three groups of firms: "strong firms" that survive the baseline COVID-19 scenario; "weak firms" that would have failed even in the absence of COVID-19 (i.e. in the non-COVID scenario); and "viable firms" that only fail if COVID-19 occurs (i.e. survive the non-COVID scenario, but fail in the baseline COVID scenario). Given that "viable" firms only fail due to COVID, we investigate to what extent these at-risk firms are similar to "strong" versus "weak" firms. We find that "viable" firms are almost identical to "strong" firms in terms of past economic performance (labor productivity and past revenue growth). These "viable" firms are failing during COVID because they are cash poor and have high leverage, metrics on which they look very similar to "weak" firms.

In response to the COVID-19 shock, governments implemented policies with broad eligibility criteria. Our framework enables us to evaluate the impact and costs of various fiscal policies. Our benchmark is a hypothetical policy that bails out only "viable" firms. The policy costs 0.77% of GDP, lowers failure rates back to their non-COVID level, and helps preserve 3.1% of private sector employment. We compare this benchmark to several interventions that mimic polices implemented in practice, including interest, tax and rent rebates, cash grants, and government guaranteed loans (or pandemic loans). We find that cash grants and pandemic loans provide the most relief, but are untargeted and costly. For example, the pandemic loan mobilizes 6.43% of GDP in government-guaranteed funding and saves 7.85% of firms and 4.02% of jobs, bringing failure rates *below* their non-COVID level.

We find that both cash grants and pandemic loans primarily save "viable" firms, but are costly because they provided substantial funding to "strong" firms. Under the pandemic loan policy, for example, 5.45% of GDP (out of a total of 6.43%) is disbursed to "strong" firms while only 0.53 and 0.45% of GDP is channeled to "viable" and "weak" firms, respectively. Of the firms saved, 56% are "viable", while the remaining 44% are "weak". We also confirm that the saved "weak" firms tend to have lower labor productivity than saved "viable" firms, suggesting that in practice, policy prevents or delays the failure of some "weak", low-productivity firms.

We are related to several papers in the literature. Khan and Thomas (2013) studies the effect of shocks originating in the financial intermediary sector. More similar to Bornstein and

Castillo-Martinez (2022), we emphasize financial frictions and liquidity shocks both at the firm and macro (sectoral) level, where macro (sectoral) shocks do not originate in the financial sector. These authors have a general equilibrium model, where they add aggregate fluctuations to the influential framework of Cooley and Quadrini (2001), who introduce financial frictions at the firm level to the firm dynamics model of Hopenhayn (1992). Relative to these papers, our model is a simple partial equilibrium model that does not micro-found the financial friction. We combine our model with detailed firm level balance sheet data that helps us capture the importance of firm level financial frictions for firm exit under large liquidity shocks originating from aggregate and sectoral demand shocks. Our contributions are that with this framework we estimate SME failure rates at the firm, sector and country levels, and provide a characterization of the surviving and failing firms under typical year and crisis scenarios.

Moreover, our financial friction is consistent with the recent literature on earnings-based constraints, wherein firms hit by liquidity shocks have difficulty borrowing from the financial sector. Empirically, Lian and Ma (2020) show that over 80% of publicly listed firm debt in the U.S. is cash flow based. More importantly for us, as we focus on SMEs who are generally private companies, Caglio et al. (2021) show that earnings based constraints are even more important for SMEs in the United States. These firms tighten their financial constraint when there is a direct hit to their earnings. Ivashina et al. (2022) show that in Spain and Peru, cash flow loans drive the contraction during the Great Financial Crisis. On the theoretical front, Drechsel (ming) shows that earnings-based constraints lead to larger business cycle amplification under shocks to cost of investment funding.

In terms of our COVID application, we also relate to several papers. Crane et al. (2022) study firm exit in the U.S. during COVID using alternative measures of exit because official measures are only available several with a few years of lag. Autor et al. (2022), study the empirical effects of "The Paycheck Protection Program (PPP)" in the U.S., which provided small businesses with roughly $800 billion dollars in uncollateralized, low-interest loans during the pandemic, almost all of which will be forgiven. Their result that the untargeted program ended up being highly regressive is consistent with our findings for European SMEs. Bartik et al. (2020) also studies the same program with a model that justifies government support based on operational delays in bank funding as the financial friction.

Our paper is structured as follows. Section 2 presents model. Section 3 introduces the Bureau van Djik Orbis firm level data for eleven European countries where we have good coverage of SMEs and reliable official data. Section 4 shows how well our framework approximates official firm failure rates in non-crisis years. Our COVID-19 application in Section 5, evaluates firm, sectoral and country vulnerability to the crisis, and assesses the cost and impact of various fiscal support measures. Section 6 presents robustness. Section 7 concludes.

2 The Model

In this section we introduce a tractable model that can be combined with firm level data to investigate the effects of liquidity shocks on firms. The model allows for a rich set of sectoral and aggregate demand and supply shocks, which can impact firm liquidity through their impact on cash flow. We focus here on the first-round, partial equilibrium effects of these shocks, emphasizing their impact on firm failure.

For each firm, we start off with economic conditions in a benchmark year, which will be informed by a large firm level dataset. Then we introduce a rich set of shocks, which are expressed as perturbations in economic conditions relative to the benchmark year. The set of shocks allow the modeler to capture a wide variety of scenarios and policy counterfactuals. In the model, firms solve a cost-minimization problem, subject to these shocks. Their optimal decisions are expressed as (non-linear) deviations from their decisions in the benchmark year.

2.1 Supply

The economy consists of $\mathcal{S}$ sectors. In each sector $s \in \mathcal{S}$ there is a mass $\mathcal{N}_s$ of firms, indexed by i. We take the initial mass of firms in each sector as given. We assume that each firm i in sector s produces according to the following sector-specific production function:

$$y_{is} = z_{is} f_s(k_{is}, A_s n_{is}, m_{is}) \tag{1}$$

In Eq. (1), y_{is} denotes gross output, k_{is} represents any fixed factor, including capital, entrepreneurial talent etc.., n_{is} is a labor input, and m_{is} denotes other variable inputs such as materials or intermediate inputs. A_s is a sector-specific labor-augmenting productivity, so that $A_s n_{is}$ is the effective labor supply in firm i, while z_{is} is a firm-specific productivity. Because the cost minimization section of our analysis is essentially static, for now we ignore time subscripts. We assume that, regardless of fixed factors, firms need both labor and intermediate goods to produce, so that $f_s(.,0,.) = f_s(.,.,0) = 0$.

We denote p_{is} as the price of output of firm i in sector s, w_s the wage rate per effective unit of labor, r_s the user cost for fixed factors and p_{ms} the price of other variable inputs. Factor prices only vary at the sector level. Prices, both for factors and output, are assumed constant in the short run, perhaps because of nominal rigidities.[6]

[6]In our COVID analysis we consider the case of flexible prices as a robustness check (Section 5). Full details on the flexible price implementation are available in Appendix E.

2.2 Demand

Each firm within a given sector sells a differentiated variety. We assume that total demand has a nested-CES structure of the form:

$$D = \left[\sum_s \mathcal{N}_s \xi_s D_s^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)} \tag{2}$$

In Eq. (2), D denotes aggregate (real) demand, D_s is sectoral (real) demand, ξ_s is a sectoral demand shock, and η is the elasticity of substitution between sectors. For simplicity, we assume that sectors are initially symmetric, and set $\mathcal{N}_s \xi_s = 1, \forall s$. We also denote with a "prime" the value of variables in the scenario under consideration, so that ξ_s is the unobserved value of the sectoral demand in sector s in the benchmark year and ξ'_s is the new value in the scenario under consideration, with $\xi'_s < \xi_s$ when demand for sector s falls and $\xi'_s > \xi_s$ when it increases.

In turn, sectoral demand D_s satisfies:

$$D_s = \left(\frac{1}{\mathcal{N}_s} \int_0^{\mathcal{N}_s} d_{is}^{(\rho_s-1)/\rho_s} \mathrm{d}i \right)^{\rho_s/(\rho_s-1)} \tag{3}$$

where ρ_s is the sector-specific elasticity of substitution between varieties.

From Eqs. (2) and (3), the demand for variety i in sector s is given by:

$$d_{is} = \xi_s^{\eta} \left(\frac{p_{is}}{P_s} \right)^{-\rho_s} \left(\frac{P_s}{P} \right)^{-\eta} D, \tag{4}$$

where P_s denotes the average sectoral price index per unit of expenditure, and P the overall price level. They satisfy:[7]

$$P_s = \left(\frac{1}{\mathcal{N}_s} \int_0^{\mathcal{N}_s} p_{is}^{1-\rho_s} \mathrm{d}i \right)^{1/(1-\rho_s)} \quad ; \quad P = \left(\sum_s \xi_s^{\eta} \mathcal{N}_s P_s^{1-\eta} \right)^{1/(1-\eta)} \tag{5}$$

Because we assume that the price of individual varieties p_{is} is constant, sectoral price indices P_s given in Eq. (5) are also constant, as long as no firm fails. The aggregate price index P, however, can change because of the demand shifters ξ_s.[8]

[7] P_s is a sectoral price index per unit of expenditure. The usual Fischer-ideal price index is given by $\mathcal{N}_s P_s$ and aggregate expenditure equals $\sum_s \mathcal{N}_s P_s D_s$.

[8] Sectoral price indices and overall CPI can also change due to "love-of-variety" effects as firms fail. We detail one way in which to adjust sectoral prices P'_s in response to firm exit in Appendix F.

We denote with a "hat" the ratio of variables relative to the benchmark period, e.g. $\hat{\xi}_s \equiv \xi'_s/\xi_s$. From Eq. (4), we can use hat algebra to express the change in demand relative to a benchmark period as:

$$\hat{d}_{is} = \hat{\xi}_s^{\eta} \hat{P}^{\eta-1} \widehat{PD} \tag{6}$$

Under the assumption that the equilibrium is symmetric in the benchmark period, $P_s \mathcal{N}_s = P \mathcal{S}^{1/(\eta-1)}$, we can write:

$$\hat{P}^{\eta-1} = \left(\frac{P'}{P}\right)^{\eta-1} = \left(\frac{\sum_s \hat{\xi}_s^{\eta} (P_s \mathcal{N}_s)^{1-\eta}}{P^{1-\eta}}\right)^{-1} = \left(\frac{1}{\mathcal{S}} \sum_s \hat{\xi}_s^{\eta}\right)^{-1}$$

Putting the two previous equations together, we obtain the following expression for the change in demand relative to a benchmark period:

$$\hat{d}_{is} = \frac{\hat{\xi}_s^{\eta}}{\sum_\sigma \hat{\xi}_\sigma^{\eta} / \mathcal{S}} \widehat{PD} \tag{7}$$

Eq. (7) indicates that the total change in sectoral demand is a function of two drivers: a relative and an aggregate one. First, sectoral demand shocks ($\hat{\xi}_s$) reallocate a given level of aggregate expenditure *across* sectors. It is the relative pattern of sectoral demand shocks that matters, not their absolute level. For instance, suppose there is no change in aggregate demand so $\widehat{PD} = 1$ and the economy consists of two sectors with $\hat{\xi}_s < \hat{\xi}_{s'}$, then $\hat{d}_s < 1 < \hat{d}_{s'}$—one sector is in recession, and the other is in a boom. The elasticity of substitution across sectors η modulates the intensity of the sectoral demand shocks ($\hat{\xi}_s$). When goods are very substitutable (high η), small sectoral demand shocks lead to large demand responses. Conversely, when demand is very inelastic (low η) demand responses become more similar across sectors (in the limit of $\eta = 0$, we obtain $\hat{d}_{is} = \widehat{PD}$). Second, for a given pattern of sectoral demand shocks, all sectors respond proportionately to changes in aggregate demand. For instance, if all sectors are affected uniformly so that $\hat{\xi}_s = \hat{\xi}, \forall s$, then Eq. (7) indicates that total demand in all sectors is affected uniformly with $\hat{d}_{is} = \widehat{PD}$.

Define $\tilde{\xi}_s^{\eta} \equiv \hat{\xi}_s^{\eta} / (\sum_\sigma \hat{\xi}_\sigma^{\eta} / \mathcal{S})$. $\tilde{\xi}_s^{\eta}$ summarizes the impact of sector-specific demand shocks on total demand and satisfies $\sum_s \tilde{\xi}_s^{\eta} / \mathcal{S} = 1$. With this notation, each firm i in sector s experiences the same proportional change in demand relative to a benchmark period, given by:

$$\hat{d}_s = \tilde{\xi}_s^{\eta} \widehat{PD} \tag{8}$$

2.3 The Firm's Cost Minimization Problem

We evaluate scenarios over a short horizon. Consequently, we assume that the prices of goods and factors are taken as given and firms meet the demand they face. Similar to Baqaee and Farhi (2020), we further assume that labor cannot reallocate across firms or sectors in the short run, so workers who cannot work for their original place of employment are laid off.

Some shocks can impose short run constraints on firms' production sets either in terms of input combinations available or in terms of productivity (A_s). For instance, as occurred during the COVID-19 pandemic, firms may be forced to reduce the size of their labor force due to health-mandated lockdowns. In order to capture such occurrences, we model the following constraint at the firm level:

$$n'_{is} \leq \hat{x}_s n_{is} \tag{9}$$

Eq. (9) captures a situation in which firms in some sectors can only employ a fraction x_{ns} of their benchmark period employment level (n_{is}). Of course a firm may decide to employ even fewer workers – for instance if demand for its goods declines significantly. It is straightforward to consider extensions to constraints on intermediate inputs m'_{is}.

Each firm minimizes variable costs by solving the following problem:

$$\begin{aligned} \min_{m'_{is}, n'_{is}} \quad & w_s n'_{is} + p_{ms} m'_{is} \\ & z_{is} f(k_{is}, A'_s n'_{is}, m'_{is}) \geq d'_{is} \\ & n'_{is} \leq \hat{x}_s n_{is} \end{aligned} \tag{10}$$

where the level of demand d'_{is} is given by Eq. (4).

We specialize the problem further by assuming that the production function $f_s(.)$ is Cobb-Douglas:

$$y_{is} = z_{is} k_{is}^{\alpha_s} (A_s n_{is})^{\beta_s} m_{is}^{\gamma_s} \tag{11}$$

where the (sector-specific) exponents α_s, β_s and γ_s sum to one.[9]

2.3.1 When Labor Input is Not Constrained

If the labor supply constraint Eq. (9) does not bind, we can solve the above program for labor and materials demand. Manipulating the first-order conditions yields:

$$\hat{m}_{is} = \hat{n}_{is} = \hat{d}_{is}^{1/(\beta_s+\gamma_s)} \hat{A}_s^{-\beta_s/(\beta_s+\gamma_s)} = \left(\tilde{\xi}_s^{\eta} \widehat{PD}\right)^{1/(\beta_s+\gamma_s)} \hat{A}_s^{-\beta_s/(\beta_s+\gamma_s)} \equiv \hat{x}_s^c \tag{12}$$

[9] Because we assume that capital k_{is} is fixed, the relevant part of this assumption is that production exhibits decreasing returns to labor and intermediate jointly, i.e. $\beta_s + \gamma_s < 1$.

Intermediate input and labor demand increase with output demand ($\tilde{\xi}_s^{\eta}\widehat{PD}$) and decrease with productivity ($\hat{A}_s$). This solution obtains as long as $\hat{n}_{is} \leq \hat{x}_{ns}$. Inputs are unconstrained as long as:

$$\hat{x}_s^c \leq \hat{x}_s \equiv \hat{x}_{ns} \tag{13}$$

We can rewrite Eq. (12) and impose Eq. (13) to get the following expression (if unconstrained):

$$\hat{x}_s^{(\beta_s+\gamma_s)}\hat{A}_s^{\beta_s} \geq \tilde{\xi}_s^{\eta}\widehat{PD} \tag{14}$$

This equation shows how supply and demand conditions help inform whether a firm is supply constrained. The left hand side of this expression captures the *supply side* of the model—the supply constraint, as well as the productivity shock. The exponent on the supply shocks is $\beta_s + \gamma_s < 1$ because adjustment in one variable input also forces an adjustment in the other one, with a total exponent $\beta_s + \gamma_s$. The right hand side captures the *demand side* of the model—the change in demand coming from sectoral or aggregate demand shifts. The inequality tells us for which firms the demand or supply side is the binding factor—demand constrains output and input use if the demand terms are lower than the supply terms, while supply constraints bind in the opposite case. Because all the variables in this expression are defined at the sectoral level, the threshold for binding supply vs. demand factors is also defined at the sectoral level.

Variable profits for unconstrained firms can be expressed as:

$$\pi'_{is} \equiv p_{is}d'_{is} - w_s n'_{is} - p_{m,s}m'_{is} = pd_{is}\left(\tilde{\xi}_s^{\eta}\widehat{PD} - (s_{ni}+s_{mi})\hat{x}_s^c\right) \tag{15}$$

where $s_{ni} = w_s n_{is}/p_{is}d_{is}$ and $s_{mi} = p_{m,s}m_{is}/p_{is}d_{is}$ denote respectively the firm's wage and material bills as a share of revenue in the period prior to the shock.[10]

2.3.2 When Labor Input is Constrained

Labor is constrained when $\hat{x}_s \equiv \hat{x}_{ns} < \hat{x}_s^c$. By manipulating the first-order conditions, we obtain:

$$\hat{n}_{is} = \hat{x}_{ns} \quad ; \quad \hat{m}_{is} = \left(\tilde{\xi}_s^{\eta}\widehat{PD}\right)^{1/\gamma_s}(\hat{A}_s\hat{x}_{ns})^{-\beta_s/\gamma_s} = \hat{x}_{ns}^{-\beta_s/\gamma_s}\hat{x}_s^{c(\beta_s+\gamma_s)/\gamma_s} > \hat{x}_{ns} \tag{16}$$

[10] If the firm is behaving competitively and optimizing over its level of output prior to the shocks, $s_{ni} = \beta_s$ and $s_{mi} = \gamma_s$, but we don't need to impose these conditions. The firm may have market power or be demand determined prior to the shock. Our framework only imposes cost-minimization during the scenario under consideration.

Compared to the unconstrained case, a binding labor supply reduces labor input and increases the use of materials. The lower is the output elasticity of materials γ_s, the stronger the response of materials when labor is constrained.

In the case of a constrained firm, variable profits are given by:

$$\pi'_{is} = p_{is} d_{is} \left(\tilde{\xi}_s^{\eta} \widehat{PD} - \hat{x}_s^c \left(s_{ni} \left(\frac{\hat{x}_{ns}}{\hat{x}_s^c} \right) + s_{mi} \left(\frac{\hat{x}_{ns}}{\hat{x}_s^c} \right)^{-\beta_s/\gamma_s} \right) \right) \tag{17}$$

Comparing this expression to Eq. (15), when labor is unconstrained, we observe that the lower use of labor tends to increase variable profits (the term $s_{ni}\hat{x}_s/\hat{x}_s^c$ decreases because $\hat{x}_{ns} < \hat{x}_s^c$), while the extra reliance on materials tends to lower profits (the term $s_{mi}(\hat{x}_{ns}/\hat{x}_s^c)^{-\beta_s/\gamma_s}$ increases). On net, and at unchanged demand, variable costs must increase when the firm is constrained. The increase in material costs is larger for firms in sectors with a relatively low output elasticity of materials (low γ_s) and a high output elasticity of labor (high β_s).

2.4 Temporary Business Shutdowns—"Mothballing"

In the case where production costs are excessive, we allow firms to prevent large falls in their cash flows by allowing them to shut down temporarily (i.e. mothballing their operations, see Bresnahan and Raff (1991)). In that case, $y'_{is} = n'_{is} = m'_{is} = \pi'_{is} = 0$. While the firm still has to cover its fixed costs and financial expenses, this option is particularly relevant for firms that face severe supply constraints that would force them to substitute—at excessively high cost—with the other available inputs. Formally a firm will choose to mothball if its variable profits are negative:

$$\pi'_{is} < 0 \Leftrightarrow \begin{cases} \tilde{\xi}_s^{\eta} \widehat{PD} < \hat{x}_s^c \left(s_{ni} \left(\frac{\hat{x}_{ns}}{\hat{x}_s^c} \right) + s_{mi} \left(\frac{\hat{x}_{ns}}{\hat{x}_s^c} \right)^{-\beta_s/\gamma_s} \right) & \text{if } \hat{x}_s^c > \hat{x}_{ns} \text{ (Labor Constrained)} \\ \tilde{\xi}_s^{\eta} \widehat{PD} < (s_{ni} + s_{mi}) \hat{x}_s^c & \text{if } \hat{x}_s^c \leq \hat{x}_{ns} \text{ (unconstrained)} \end{cases} \tag{18}$$

For constrained firms, direct inspection of Eq. (18), reveals that mothballing is more likely when labor supply is constrained and firms have a low materials output elasticity γ_s relative to the labor output elasticity β_s.

For unconstrained firms we can substitute $\hat{x}_s^c$ using Eq. (12) to get the following expression in terms of shocks:

$$\hat{A}_s^{\beta_s} < (s_{ni} + s_{mi})^{\beta_s + \gamma_s} \left(\tilde{\xi}_s^{\eta} \widehat{PD} \right)^{1 - \beta_s - \gamma_s} \tag{19}$$

Eq. (19) shows that when $\beta_s + \gamma_s < 1$ (i.e. there are diminishing returns to variable inputs), unconstrained firms will shut down when total demand ($\tilde{\xi}_s^{\eta}\widehat{PD}$) is excessively *high* or productivity is low.

2.5 Evaluating Business Failures

To evaluate business failure, we assume that firms follow a simple decision rule—they remain in business as long as their initial cash balances and operating cash flow over a given assessment period are sufficient to cover their financial expenses. Otherwise, they fail.

In the remainder of this section, we formalize this liquidity based failure criterion. We begin by showing how to link the expressions for variable profits in the scenario under consideration (π'_{is}) to firm cash flow in that scenario (CF'_{is}). We then discuss how to use firm cash flow to evaluate whether a firm is illiquid. In the process, we also address two complications. First; how to deal with important missing variables in typical balance sheet data—such as fixed costs or taxes and second; how to apply our framework in a mixed frequency context—such as when firm balance sheet data is available at an annual frequency, but shocks are measurable at a higher frequency.

We start by defining cash flow in some period t as:

$$CF_{is} \equiv p_{is}d_{is} - w_s n_{is,t} - p_{m,s}m_{is} - F_{is} - T_{is} + B_{is} - R_{is} = \pi_{is} - F_{is} - T_{is} + B_{is} - R_{is} \tag{20}$$

where pd_{is} represents revenues, $w_s n_{is}$ represents wages, and $p_{m,s}m_{is}$ the intermediate input bill. F_{is} represents any costs associated with fixed factors (rent, utilities, etc.), including capital costs ($r_s k_{is}$), T_{is} denotes business taxes, B_{is} denotes new borrowing and R_{is} repayments of principal on existing debt. The last expression writes operating cash flow in terms of the variable profits (π_{is}), minus payments to fixed factors and taxes and the change in debt. Interest payments on debt will be consider later in this section.

Cash flow in the scenario under consideration is therefore:

$$CF'_{is} = \pi'_{is} - F'_{is} - T'_{is} + B'_{is} - R'_{is}$$

When financial markets function normally, firms typically opt to roll over their existing debt paying only the interest. This suggests that R_{is} is often 0. As we don't observe debt repayments, we calculate cash flow assuming that no repayments *are required* (but are optional for firms if they so choose). Therefore, to assess whether a firm is sufficiently illiquid that it fails, all we need to know is whether that firms' cash flow *absent any debt repayment* is sufficient to cover their financial expenses. Hence we impose $R'_{is} = R_{is} = 0$.

As long as fixed costs and taxes are unchanged between the benchmark year and the scenario under consideration ($F'_{is} = F_{is}$ and $T'_{is} = T_{is}$), we can difference them out by considering the change in cash flows from CF_{is} to CF'_{is} (i.e. from the observed to the predicted cash flows).[11] An advantage of this approach is that it does not require information on fixed cost or taxes in the benchmark year, which may not always be available in balance sheet data.

$$\begin{aligned} CF'_{is} &= \pi'_{is} - F_{is} - T_{is} + B'_{is} \\ &= CF_{is} + (\pi'_{is} - \pi_{is}) + (B'_{is} - B_{is}) \end{aligned} \tag{21}$$

The predicted cash flow (CF'_{is}) is then obtained by substituting our estimated variable profits π'_{is} using Eqs. (15) or (17) depending on whether the firm is unconstrained, labor constrained or material constrained. All that remains is to make an assumption on new borrowing $B'_{is} - B_{is}$. This can be done based on explicit criteria (e.g. all firms can borrow x% of their tangible fixed assets) or implicitly by allowing *temporary* credit lines that must be repaid at prespecified points in time. In our analysis we use both approaches and detail how to implement temporary credit lines later in this section.

We consider two types of explicit borrowing constraints based on the level of capital or pre-shock earnings:

$$B'_{is} \leq \alpha_K \times k_{is} \tag{22}$$

$$B'_{is} \leq \alpha_E \times \pi_{is} \tag{23}$$

where α_K and α_E represent the fraction of capital or pre-shock earnings available to borrow.[12]

Next, when implementing the framework, it may happen that balance sheet data is available at one frequency (e.g. annual) and shocks at another (e.g. weekly). To account for this, we let the time period t be denoted by a touple $t = (y, \tau)$ where $y \in \mathcal{Y} \equiv \{y_1, \ldots, y_n\}$ denotes years and $\tau \in \mathcal{T} \equiv \{1, \ldots, \bar{T}\}$ denotes subperiods within each year (e.g. weeks, months, quarters). In the general case, the cash flow condition becomes:

$$CF'_{is,y,\tau} = \frac{CF_{is,y_0}}{\bar{T}} + \left(\pi'_{is,y,\tau} - \frac{\pi_{is,y_0}}{\bar{T}}\right) + (B'_{is,y,\tau} - B_{is,y,\tau}) \tag{24}$$

where CF_{is,y_0} and π_{is,y_0} represent annual cash flow and profits from the benchmark year, respectively; and $\pi'_{is,y,\tau}$ represents profits in the scenario under consideration in year y and

[11] For short horizons such as one year, this is likely a reasonable assumption. Rental contracts often fix rent for several years, and many business taxes are paid in the following calendar year. Therefore, from a liquidity perspective the taxes a business needs to pay in year t are likely determined in year $t-1$ and will not change if an unexpected shock occurs in year t.

[12] In the pre-shock data, some firms may already have borrowing levels that exceed these constraints. We impose $B_{is'} \leq B_{is}$ for these firms so that they are not forced to contract their borrowing in response to the imposition of these borrowing constraints.

subperiod τ, given by:

$$\pi'_{is,y,\tau} = \begin{cases} \frac{p_{is}d_{is,y_0}}{\bar{T}} \left(\tilde{\xi}^{\eta}_{s,y,\tau} \widehat{PD}_{y,\tau} \right. \\ \qquad \left. -\hat{x}^c_{s,y,\tau} \left(s_{ni,y_0} \left(\frac{\hat{x}_{ns,y,\tau}}{\hat{x}^c_{s,y,\tau}} \right) + s_{mi,y_0} \left(\frac{\hat{x}_{ns,y,\tau}}{\hat{x}^c_{s,y,\tau}} \right)^{-\frac{\beta_s}{\gamma_s}} \right) \right) & \begin{array}{c}\text{if } \hat{x}^c_{s,y,\tau} > \hat{x}_{ns,y,\tau} \\ \text{(Labor Constrained)}\end{array} \\ \frac{p_{is}d_{is,y_0}}{\bar{T}} \left(\tilde{\xi}^{\eta}_{s,y,\tau} \widehat{PD} - (s_{ni,y_0} + s_{mi,y_0}) \hat{x}^c_{ns,y,\tau} \right) & \begin{array}{c}\text{if } \hat{x}^c_{s,y,\tau} \leq \hat{x}_{s,y,\tau} \\ \text{(unconstrained)}\end{array} \end{cases} \tag{25}$$

Next, denote initial (benchmark year) cash balances $\mathcal{Z}_{is,y_0}$ and annual financial expenses, defined as interest payments due on the firms' debt, $\iota L_{is,y_0}$. Let $\mathcal{T}_i \subset \mathcal{T}$ denote the subperiods within the year when interest payments are due. Then, define the cash position in each period $t = (y, \tau)$ of the scenario as:

$$\begin{aligned} \mathcal{Z}_{is,y,\tau} = \mathcal{Z}_{is,0,0} &+ \sum_{y'<y} \sum_{\tau' \leq \bar{T}} \left(CF'_{is,y',\tau'} - \iota L_{is,y_0} \frac{1}{|\mathcal{T}_i|} \mathbb{1}_{\tau' \in \mathcal{T}_i} \right) \\ &+ \sum_{\tau' \leq \tau} \left(CF'_{is,y,\tau'} - \iota L_{is,y_0} \frac{1}{|\mathcal{T}_i|} \mathbb{1}_{\tau' \in \mathcal{T}_i} \right) \end{aligned} \tag{26}$$

where $|\mathcal{T}_i|$ represents the size of the set $\mathcal{T}_i$. Note that allowing for the set $\mathcal{T}_i$ to differ from $\mathcal{T}$ allows for interest payments to occur at a lower frequency than shocks.

Finally, let $\mathcal{F} = (Y_f, T_f) \subset (\mathcal{Y}, \mathcal{T})$ denote a set of "assessment periods" where firm failures are assessed. Firms survive if:

$$\mathcal{Z}_{is,y,\tau} \geq 0, \quad \forall (y, \tau) \in \mathcal{F} \tag{27}$$

and fail otherwise.

Note that in all of our scenarios, we consider a single year comprised of 52 weeks (i.e. $y_n = 1$ and $\bar{T} = 52$). If a weekly assessment period is chosen ($\mathcal{F} = \{(1, \tau)\}_{\tau=1}^{52}$), then firms need positive cashflow in every week of the year to survive.[13] In contrast, if an annual assessment period is chosen ($\mathcal{F} = (1, 52)$), firms can experience temporary periods of illiquidity within the year and only fail if they cannot cover their financial expenses by the end of the year.

Within our model, infrequent assessment periods have the interpretation of temporary credit lines with zero interest rates that have to be repaid by the next assessment period.[14]

As such, the annual assessment period assumed in our baseline implementation captures the ability of SMEs to take advantage of these options without explicitly modeling them, while also capturing that they cannot be used indefinitely. When implementing these temporary

[13] In our model, aggregate demand is constant within the year. When considering multiple assessment periods, this means demand is reallocated from failing firms to surviving ones. Appendix F discusses how this is implemented. We are grateful to an anonymous referee for pointing out the importance of this adjustment.

[14] This method implicitly accounts for other actions firms could take such as delaying the payment of receivables or running down input inventories.

credit lines we set $B'_{is,y,\tau} - B_{is,y,\tau} = 0, \;\; \forall(y,\tau)$.

Two caveats are worth noting regarding our failure criterion. First, while Eq. (26) has the advantage of simplicity, it assumes that firms with a cash flow shortfall in any assessment period $(y,\tau) \in \mathcal{F}$ cannot access credit markets to borrow new funds. Note however, that this criterion also assumes that existing debt levels can be maintained, but that firms are constrained in each assessment period $(y,\tau) \in \mathcal{F}$ when it comes to obtaining *additional* funds. This is not unrealistic for SMEs as shown in Caglio et al. (2021).

A second caveat is that we ignore the role of bankruptcy courts. In theory, as long as a business remains viable, the failure to repay creditors in the short run does not mean that it ceases to operate. Instead, business liabilities should optimally be restructured under bankruptcy proceedings. In practice, however, there is substantial variation in bankruptcy regimes across countries. In the U.S. for example, there is automatic stay and lenders lend based on future cash flow during the restructuring process. However, this is mostly for the larger corporations, but it is less well suited for SMEs. Moreover, bankruptcy courts in many countries may not be able to efficiently preserve viable businesses in the middle of a large downturn if a wave of small business failures congests the courts.

3 Taking the Model to the Data

To bring the model to the data, we construct empirical counterparts to the sector-specific ($\tilde{\xi}^{\eta}_{s,y,\tau}$) and aggregate ($\widehat{PD}_{y,\tau}$) demand shocks, and sectoral supply ($\hat{x}_{ns,y,\tau}$) and productivity ($\hat{A}_{s,y,\tau}$) shocks.[15] We also estimate sector-specific output elasticities (β_s, γ_s). Together with benchmark year, firm level factor shares ($s_{n,is,y_0}, s_{m,is,y_0}$) and sales ($p_{is,y_0} d_{is,y_0}$), we construct a counterfactual change in cash flow. With data on the firm's cash balances ($\mathcal{Z}_{is,y_0}$), financial expenses ($\iota L_{is,y_0}$) and cash flow (CF_{is,y_0}) in the benchmark year, we then evaluate Eq. (26) to determine which businesses fail and when.

Because the construction of shocks varies based on the specific application, we defer the details of shock construction to the sections describing each application. Our source for the firm level data is common to all applications.

Firm Level Data: We use Orbis, a firm level data set from BvD-Moody's that covers both private and publicly listed firms. Orbis data are collected by BvD from various sources, including national business registries, and are harmonized into an internationally comparable format. The Orbis database covers more than 200 countries and over 200 million firms. The

[15] Note that because we directly assess the change in sectoral demand according to Eq. (7), and not the underlying shock to preferences $\tilde{\xi}_{s,y,\tau}$, we do not need to make an assumption about the elasticity of substitution η. This is already encoded in our measure of $\tilde{\xi}^{\eta}_{s,y,\tau}$.

longitudinal dimension and representativeness of Orbis data vary from country to country, depending on which firms are required to file information with business registries.

In our analysis, we focus on a set of eleven countries. The countries included are Czech Republic, Finland, France, Hungary, Italy, Poland, Portugal, Romania, Slovak Republic, Slovenia and Spain. As described in Table 16 in the appendix, we have good coverage of aggregate revenues for the countries in our sample, both for all firms and SMEs.

We evaluate SME failures because these firms account of a large fraction of economic activity and are particularly vulnerable to liquidity shocks. Across our sample of countries, SMEs account for 62.68% of employment, 61.34% of payroll, 65.52% of revenue, and 65.90% of total assets.[16] These SMEs are especially exposed to liquidity shocks because they tend to have lower cash balances, be bank-dependent, and have limited ability to draw on credit lines.

We use data on firm revenue, wage bill, material cost, number of employees, net income, depreciation, cash balance and financial expenses.[17] Cash flow is calculated as the sum of net income and depreciation, less financial profits. The analysis focuses on non-financial SMEs.[18]

Estimating Output Elasticities: We also use Orbis data to estimate labor and material elasticities (β_s and γ_s) at the 2-digit NACE level for each country. Taking into account our modeling assumption that labor and intermediate inputs are variable inputs, and recent critiques of the key identifying assumptions of popular production function estimation techniques, we estimate elasticities as the weighted average of the firm revenue share of input expenditures (e.g., labor cost share of revenue and material cost share of revenue), where the weights are given by firm revenue.[19] Due to the lack of price data, the elasticities we estimate are revenue, rather than output, elasticities. The mean and standard deviation of the labor and material elasticities are reported in Table 15.

[16]SME shares are based on the cleaned Orbis data used in analysis. Aggregation is done over our sample of countries. The SME shares are first calculated at the country level and aggregated across countries using country GVA for weighting. The contribution of SMEs to the aggregate economy in the official data mimics the numbers here based on Orbis, as shown in detail in Kalemli-Ozcan et al. (2019).

[17]We winsorize all of the level variables used for analysis at the 99.9th percentile. Note also that, in principle, inital cash balances $\mathcal{Z}_{is}$ could include overdraft facilities or undrawn credit lines. Unfortunately, Orbis does not contain any information on these so we use intial cash balances for $\mathcal{Z}_{is}$ and present several exercises where we allow for firms to access additional funds until the end of the year.

[18]Additional data construction details: we focus on firms in NACE 1-digit sectors A, B, C, D, E, F, G, H, I, J, L, M, N, P, Q, R, S. We impose standard cleaning steps that check the internal consistency of balance sheet data. We exclude firms that do not report on the line items needed in order to calculate total assets and total liabilities. We exclude financial and insurance activities (K), public administration and defense (O), activities of households as employers (T), and activities of extraterrestrial organizations and bodies (U). We also exclude sub-sectors 78 and 81 in the Administration (N) because they have very large labor cost shares which together with our labor constraint generates unrealistically high failure rates and cash shortfalls. We exclude companies owned by public authorities and firms that were previously recorded by Orbis as bankrupt, dissolved, or illiquid.

[19]See Ackerberg et al. (2015), Gandhi et al. (2012), Levinsohn and Petrin (2003), and Wooldridge (2009). Our approach is similar to that of Blackwood et al. (2021) for variable inputs and is an alternative to the parametric approach of Gandhi et al. (2012).

4 Applying the Framework to Typical Years

With our first application, we show that our framework produces failure rates in line with the observed data, and that the characteristics of failing firms are consistent with findings in the literature. Specifically, we consider a scenario that is equivalent to a one year ahead forecast of firm failure rates in our sample of countries, wherein we define benchmark years as 2016-2018 and predict firm failures in 2017-2019. We refer to these scenarios as a "typical year" scenarios. We then compare our predicted failure rates to those obtained from official sources, and compare the characteristics of failing firms to those emphasized in the existing literature.

Calibrating Shocks: We calibrate shocks using data from OECD.Stat and Eurostat to measure the perturbations in economic conditions around each benchmark year. This means, for example, that when we forecast firm failures in 2017, we measure shocks as changes in economic conditions between 2016 and 2017. All sectoral shocks are measured at the one-digit NACE level, which is the finest level of granularity for which data are consistently available across sectors in our sample of countries.

Fig. 8 depicts the average total demand and sectoral productivity shocks across countries between 2017 and 2019. The total demand shock is composed of aggregate demand and sector-specific demand shocks. The aggregate demand shock ($\widehat{PD}$) is measured as the cumulative quarterly change in real GDP in each country. The sector-specfic demand shock ($\tilde{\xi}_s^{\eta}$) is constructed by first obtaining annual sectoral revenue growth for each country, and then normalizing the revenue growth to be consistent with aggregate demand, Eq. (7), by constructing $\tilde{\xi}_s^{\eta} = \hat{\xi}_s^{\eta} / (\sum_{\sigma} \hat{\xi}_{\sigma}^{\eta} / \mathcal{S})$. This ensures that these sectoral demand shocks are reallocative – i.e. lowering one sectors $\tilde{\xi}_s$ redistributes that lost demand to all other sectors. The sectoral productivity shock ($\hat{A}_s$) is measured as the annual growth in output per worker for each country. Finally, we assume the input constraints are inactive ($\hat{x}_{ns} = \hat{x}_{ms} = \infty$) because there were no notable supply bottlenecks or labor market disruptions in our set of countries during the years under consideration.

Forecasting Firm Failures: We use firm level data from Orbis in each benchmark year to forecast firm failures. Starting with the benchmark year cash position of each firm, we use our model equations and calibrated shocks to simulate each firm's cash flow over the subsequent year. We combine the estimated cash flow with our liquidity criterion (Eq. (26)) to predict which firms fail. Our "typical year" scenario makes three assumptions, described in Section 2. We estimate weekly cash flow in order to exploit within-year variation in shocks, but evaluate the liquidity criterion at the end of each year to allow firms to smooth cash flow during the year. We allow firms to temporarily mothball in periods of low profitability. And we assume firms can maintain existing debt levels, but must pay the interest due on this debt monthly.

One limitation of this analysis is that we abstract from idiosyncratic shocks. This is not because they are not important, but simply because we cannot measure them. In Appendix D we present some evidence that country-sector pairs where cash flow is less predictable year over year tend to be the sectors where our exercise delivers the largest forecast errors. This suggests that with more granular measurement of shocks, our approach could obtain even better forecast performance.

Comparing Forecasted versus Official Failure Rates: Fig. 1, Fig. 2, and Table 1 show that the "typical year" implementation of our framework produces failure rates broadly in line with the observed data. Fig. 1 pools all countries over the 2017-2019 period, and shows the full distribution of forecast errors (estimated - actual failure rates) at the 1-digit sector level. Overall, 80% of our forecast errors are within four percentage points of the true value, with a mean forecast error of 0.69 percentage points (pp) and mean absolute error of 2.29 pp.[20] Note, the bulk of the extreme forecast errors come from Portugal and Romania, which is confirmed in Table 1. Dropping these two countries lowers our mean forecast error to 0.45 pp and the mean absolute error to 2.12 pp. Moreover, Table 1 shows that outside of Portugal and Romania, our framework captures the cross-country variation in failure rates well, with a forecast error of less than one pp in over half of our sample.[21] Fig. 2 further shows that our framework matches the sectoral patterns in failure rates with reasonable accuracy.

It is worth noting that our framework can, in theory, accommodate very granular shocks—down to the firm level. For this exercise, we used the most consistently reliable sectoral shock data, at the finest level of granularity available—1-digit sector level. Yet, despite the use of less granular shocks, our framework captures well the average pattern of failure rates at both the country and sector levels.

[20] Average failure rates for our sample of countries over this period is 8.94% suggesting a moderate bias of 7.7% (0.69/8.94). All averages are weighted using (country x sector x year) GVA.

[21] Table 17 in Appendix B reports the forecast errors at the one-digit sector level.

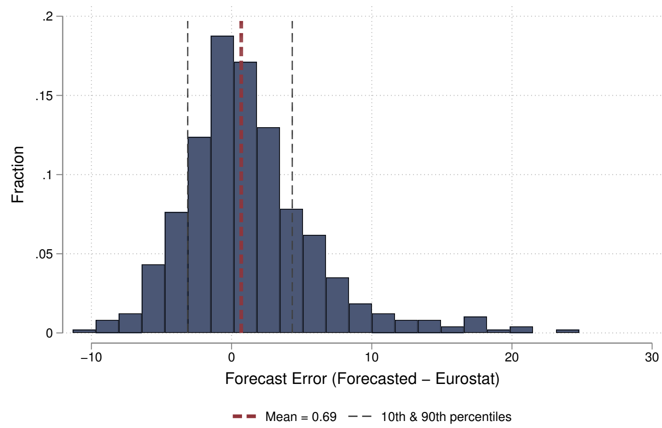


Figure 1: Forecast Errors at the Country x Sector Level (2017-2019).
Notes: Eurostat failure rates are obtained from the Structural Business Statistics for employer businesses at the (country x 1-digit NACE x year) level. Failure rates are forecasted by combining Orbis firm level balance sheet data with sector-specific demand and labor productivity shocks calculated using Eurostat national accounts at the 1-digit NACE level, and aggregate demand shocks measured as quarterly GDP growth from OECD.Stat. The liquidity criterion is evaluated for each firm at the end of the year. This histogram shows the distribution of forecast errors at the (country x 1-digit NACE sector x year) level.

Table 1: Failure Rates Comparison at the Country Level (2017-2019)

	Eurostat Failure Rate	Forecasted Failure Rate	Δ (Forecasted - Eurostat)
Czech Republic	7.46	7.46	0.01
Finland	10.62	10.07	-0.55
France	9.84	9.46	-0.38
Hungary	9.86	9.48	-0.39
Italy	7.41	9.77	2.36
Poland	12.49	11.94	-0.55
Portugal	7.69	13.08	5.39
Romania	8.65	13.24	4.59
Slovak Republic	9.19	10.05	0.86
Slovenia	8.63	6.89	-1.74
Spain	8.36	8.32	-0.04
Weighted Average	8.97	9.67	0.69

Notes: Eurostat failure rates are obtained from the Structural Business Statistics for employer businesses at the (country x 1-digit NACE x year) level. Failure rates are forecasted by combining Orbis firm level balance sheet data with sector-specific demand and labor productivity shocks calculated using Eurostat national accounts at the 1-digit NACE level, and aggregate demand shocks measured as quarterly GDP growth from OECD.Stat. The liquidity criterion is evaluated for each firm at the end of the year. The table shows (1) official Eurostat and (2) forecasted failure rates, as well as the (3) the forecast error (i.e. Forecasted-Eurostat failure rate) at the country level. The (country x sector x year) observations are first aggregated to the (county x year) level using sectoral GVA as weights. The observations are then aggregated to the country level by taking a simple average over time (2017-2019). The cross-country average is calculated using GVA for weighting.

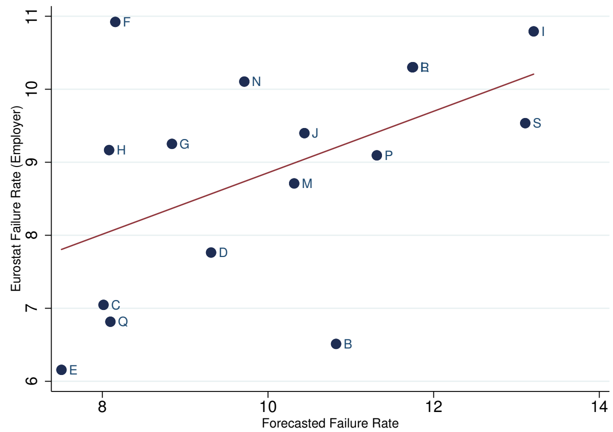
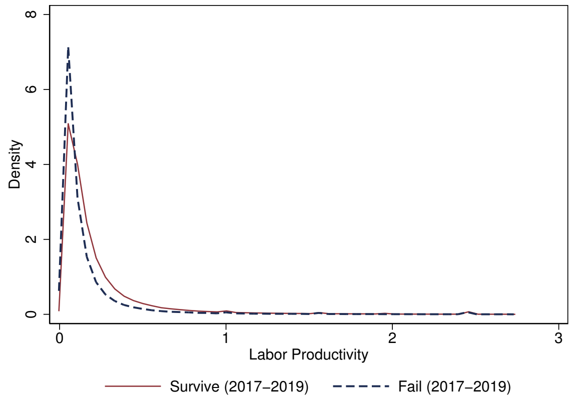


Figure 2: Forecasted versus Actual Failure Rates at the Sector Level (2017-2019).
Notes: Eurostat failure rates are obtained from the Structural Business Statistics for employer businesses at the (country x sector x year) level. Failure rates are forecasted by combining Orbis firm level balance sheet data with sector-specific demand and labor productivity shocks calculated using Eurostat national accounts at the 1-digit NACE level, and aggregate demand shocks measured as quarterly GDP growth from OECD.Stat. The liquidity criterion is evaluated for each firm at the end of the year. The figure plots official Eurostat versus estimated failure rates at the sector level. The (country x sector x year) observations are first aggregated to the (sector x year) level using cross-country sectoral GVA as weights. The observations are then aggregated to the sector level by taking a simple average over time (2017-2019).

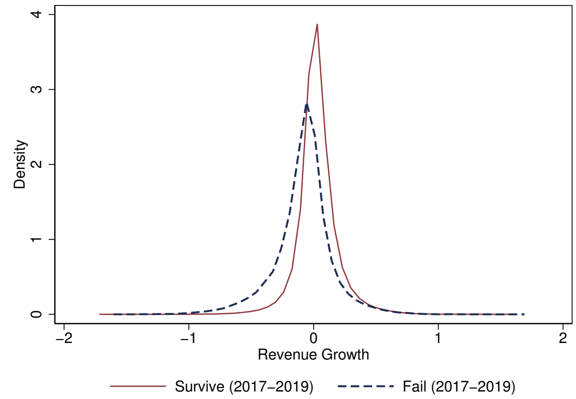
Characterizing Failing and Surviving Firms: An advantage of our framework is that we can investigate differences in firm characteristics between firms predicted to fail and those predicted to survive in any given year. Fig. 3 compares the distributions of labor productivity, past revenue growth, initial cash-to-assets ratio, and short-term leverage for failing and surviving firms in 2017-2019. First, given our liquidity based criterion, we find that firms with relatively low cash-to-assets ratios and high leverage are predicted to fail. We also find that failing firms tend to have lower labor productivity and growth.

The weakness of failing firms is further investigated in Table 2, where firms predicted to fail are reported to be on average smaller in terms of revenue and employment and younger than surviving firms. Moreover, firms predicted to fail are those that shrank and were unprofitable in previous periods. Taken together, these findings suggest that our liquidity criterion matches stylized data facts and predictions of firm dynamics models regarding exiting firms.[22] Our findings also match the differences in firm performance between failing and surviving firms — a difference often used to justify solvency based failure criteria in firm dynamics models.

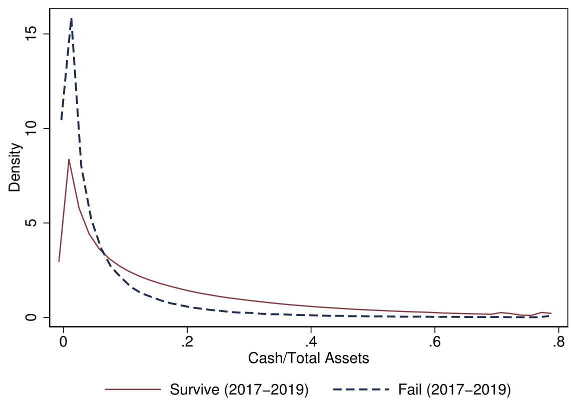
[22]See Albuquerque and Hopenhayn (2004); Arellano et al. (2019); Ayres and Raveendranathan (2021); Cooley and Quadrini (2001); Foster et al. (2016); Lee and Mukoyama (2015); Tian (2018).



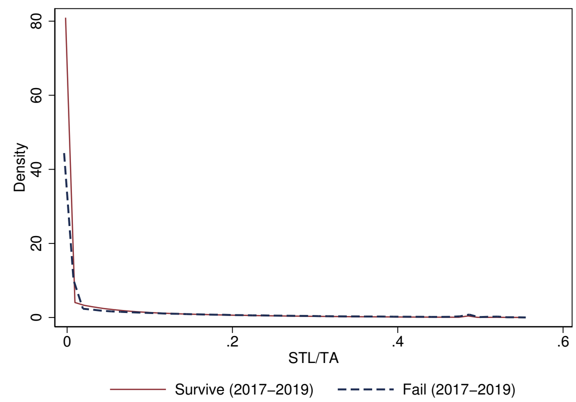
(a) Log Labor Productivity



(b) Revenue Growth Rate (3-year Average, %)



(c) Cash/Total Assets (%)



(d) Short-term Leverage (%)

Figure 3: Distributions of Survivors vs. Failures (2017-2019)

Notes: Depicted are the distributions of (a) log labor productivity (sales per worker), (b) revenue growth rate in percent, (c) beginning of period cash-to-total assets ratio and (d) short-term leverage (defined as short-term loans/initial total assets) of firms who we predict will survive or fail in 2017-2019. Note that a firm that fails in 2019 will be classified as surviving in 2017 and 2018 and as failing in 2019.

Table 2: Summary Statistics (Median): Survivors vs. Failures (2017-2019)

	Survive	Fail
Cash/Total Assets	10.35	2.15
EBITDA/Total Assets	9.50	-11.21
Employment Growth	0.55	-0.53
Firm Age	14.46	12.21
Labor Productivity	0.15	0.10
Number of Employees	11.31	8.88
Revenue (Millions USD)	1.83	1.11
Revenue Growth	1.70	-5.89
Short-term Loans/Total Assets	0.86	0.92

Notes: Table reports the median of variables of interest, separately for firms who we predict will (1) survive or (2) fail in 2017-2019. Note that a firm that fails in 2019 will be classified as surviving in 2017 and 2018 and as failing in 2019.

5 Applying the Framework to COVID-19

During economic crises, two concerns often prevail—whether productive firms can survive without government intervention and whether government support will save productive, or instead, already weak firms. With our approach, we can address both concerns directly. To illustrate how our framework can be applied in a crisis context, we use COVID-19 as a laboratory. The COVID-19 crisis is the perfect setting to implement our framework because the combination of an unprecedented reallocation of demand across sectors and severe lockdowns and put enormous pressure on firms' cash flows and many firms' liquidity. This forced governments to react swiftly with policies that disbursed funds to struggling firms. We first describe our calibration of sectoral and aggregate shocks. We then evaluate how vulnerable firms in our set of countries were to COVID-19 shocks, and describe the characteristics of firms predicted to fail. Finally, we evaluate the cost and impact of policy support.

5.1 Calibrating Shocks for a Counterfactual 2020 (non-COVID)

In order to understand the effects of COVID on firm exit, we need to have an estimate of what 2020 might have looked like had COVID never occurred. The proportion of firms that fail and their size, employment and other characteristics can inform us about the types of firms one might typically expect to fail vs. those that we expect could have failed in COVID (absent government support) and those that who remain liquid because of government support. Our approach is to assume that 2020 failure rates would have been identical to those in 2019—which, because at the time of writing the data on *2019* failure rates is still provisional, we implement our framework in an identical manner as in Section 4.

Specifically, we start with firm balance sheet data from Orbis in 2018. We then use data on the change in aggregate demand $\widehat{PD}$, sector-specific demand $(\tilde{\xi}_s^{\eta})$ and sectoral productivity $(\hat{A}_s)$ to simulate 2019. The aggregate demand shock $(\widehat{PD})$ is measured at the country level as the cumulative quarterly change in real GDP over 2018-2019. All sector-specific shocks are measured at the 1-digit NACE sector level because of data availability. Specifically, the sector-specific demand shock $(\tilde{\xi}_s^{\eta})$ is measured at the country-sector level using annual revenue growth in each sector, normalized to be consistent with aggregate demand and to function as a reallocative demand shock $(\tilde{\xi}_s^{\eta} = \hat{\xi}^{\eta}/(\sum_{\sigma} \hat{\xi}_{\sigma}^{\eta}/S))$. The sectoral productivity shock ($\hat{A}_s$ is measured at the country-sector level using annual growth in output per worker. We further assume that the labor supply constraint is inactive $\hat{x}_{ns} = \infty$.

Using the end of 2018 cash position of each firm as the beginning of 2019 cash position, we simulate 2019 with the above shocks active and assess at the end of 52 weeks whether a firm has a positive end of 2019 cash balance.[23] If it does not, that firm is assumed to exit. The distribution of exiting and surviving firms in this simulation of 2019 is then used as our estimate of the distribution of firms that would survive/exit in a counterfactual non-COVID 2020 which henceforth we refer to as our "non-COVID scenario".

5.2 Calibrating COVID-19 Shocks

In our COVID-19 scenarios, we define shocks as perturbations in economic conditions caused specifically by the COVID-19 pandemic, relative to conditions in a benchmark year. In order to highlight how our framework can be deployed quickly at the onset of a crisis, we calibrate our shocks using information available at the early stages of the COVID-19 crisis—June 2020. Importantly we measure shocks at the *4-digit* NACE sector level to capture the extreme heterogeneity that COVID had on firms in different sectors of the economy.[24]

Essential versus Non-essential Sectors: We first separate sectors, at the 4-digit NACE level, into essential and non-essential, based on the U.S. Department of Homeland Security Guidance on the Essential Critical Infrastructure Workforce.[25] While the DHS does not provide a list of industry codes that are deemed to be essential, we classify sectors based on the information provided regarding the types of workers and activities considered as part of essential critical infrastructure. Among essential workers are those working in public health, public safety, food supply chain, energy infrastructure, transportation and logistics, critical manufacturing, hygiene products and services, among others.

Sectoral Input Shock: In the context of COVID-19, an important constraint facing firms was that workplace restrictions limited the number of workers that could be used on site. Because in the benchmark (pre-COVID) year, the labor supply constraint was inactive, the sectoral labor supply shock ($\hat{x}_s$) captures by how much firms are forced to reduce their labor force due to lockdowns and workplace social distancing requirements.

To calibrate the labor supply shock, we follow Dingel and Neiman (2020) and measure the feasibility of remote work by industry. To construct the measure, we start with the "work context" and "generalized work activities" surveys conducted by the Occupational Information Network (O*NET). We classify occupations into those that can be performed remotely versus those that cannot, based on characteristics such as reliance on being outdoors, interacting with patients, repairing and inspecting structures and equipment, controlling machines, handling and moving objects, among others. We then use information from the U.S. Bureau of Labor

[23]Firms are allowed to temporarily close ("mothball") if this leads to higher variable profits in that week.

[24]Appendix D.2 details the effects on our results of using less granular shocks (aggregated at the 3-digit, 2-digit and 1-digit level.

[25]See CISA's Guidance on the Essential Critical Infrastructure Workforce.

Statistics (BLS) on the prevalence of each occupation by NAICS industry. Using a cross-walk between NAICS and NACE codes, we arrive at the fraction of employees that can perform their work remotely by 4-digit NACE industry.

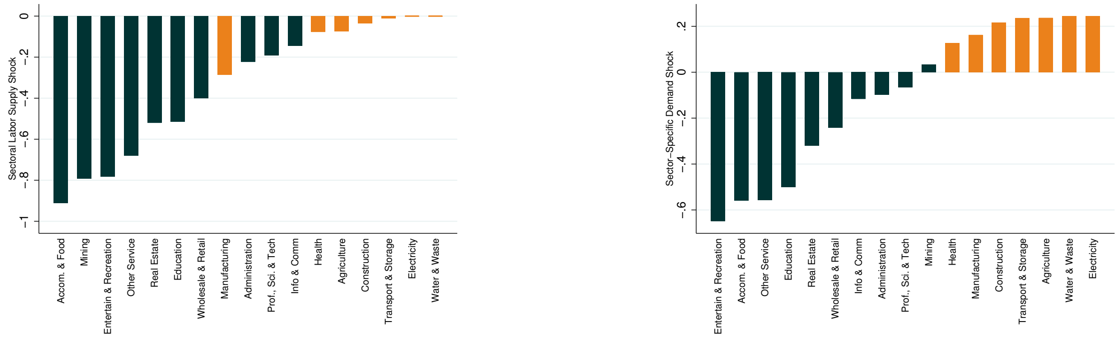
In constructing the sectoral labor supply shock ($\hat{x}_s$), we assume that firms in non-essential sectors can produce with at most the fraction of workers they can shift to remote work, and that firms in essential sectors face no such restriction. The left panel of Section 5.2 illustrates the severity of the labor supply shock at the 1-digit NACE level.[26] The Accommodation & Food Service and Arts, Entertainment & Recreation sectors are among the most affected, while essential infrastructure sectors, including Electricity and Water & Waste, remain largely unaffected.

Sector-Specific Demand Shock: The sector-specific demand shock ($\tilde{\xi}_s^{\eta}$) measures how much the COVID-19 pandemic reallocates demand across sectors, relative to a benchmark (pre-COVID) year. Because the pandemic affected the ability and willingness of consumers to interact in person, we calibrate the shock using information on whether industries are customer facing. Specifically, using O*NET surveys, we classify occupations based upon reliance on face-to-face interactions. We consider occupations as highly reliant on face-to-face interactions when working with external customers or in physical proximity, caring for others, working with the public, and selling to others are deemed important. As with the sectoral labor supply shock, we aggregate occupation-level data to arrive at an estimate of the fraction of employees reliant on face-to-face interactions at the 4-digit NACE level.

We assume that under COVID-19 the raw sector-specific demand shock ($\hat{\xi}_s^{\eta}$) is one in essential sectors and one minus the fraction of customer facing employees in non-essential industries. We then normalize the raw sectoral demand shocks to be consistent with aggregate demand, Eq. (7), by constructing $\tilde{\xi}_s^{\eta} = \hat{\xi}_s^{\eta} / (\sum_{\sigma} \hat{\xi}_{\sigma}^{\eta} / \mathcal{S})$. Recall that this adjustment makes these sectoral demand shocks reallocative such that even with modestly negative aggregate demand shocks, large dispersal in sectoral demand shocks can lead demand to rise in some sectors. The right panel of Section 5.2 illustrates the size of the sector-specific demand shock at the 1-digit NACE level. The figure illustrates that COVID-19 reallocates expenditure from highly affected non-essential sectors such as Arts, Entertainment, & Recreation to non-affected essential sectors including Water & Waste.[27]

[26]To construct Section 5.2, we aggregate to the 1-digit level by first averaging 4-digit NACE shocks to the 1-digit level in each country and then using the GVA sector share of each country to aggregate 1-digit sector shocks across countries.

[27]Within each country $\sum_s \tilde{\xi}_s^{\eta} / \mathcal{S} = 1$ holds. However, Section 5.2 aggregates sector-specific demand shocks at the 1-digit NACE level across countries using the gross value added sector share of each country. Consequently, the sector-specific demand shocks depicted in the figure do not sum to one.



(a) Sectoral Labor Supply Shock

(b) Sector-Specific Demand Shock

Figure 4: Shocks by Sector: Baseline COVID-19 Scenario

Notes: Depicts the COVID-19 (a) sectoral labor supply and (b) sector-specific demand shocks by 1-digit NACE sector. Shocks are first aggregate from the 4-digit NACE to 1-digit NACE level by taking a simple average across 4-digit sectors within each country. The GVA sector share of each country is used to aggregate 1-digit sector shocks across countries. Sectors composed mainly of non-essential industries are depicted in blue and those composed mainly of essential industries are depicted in orange

Aggregate Demand Shock: The aggregate demand shock measures the change in aggregate expenditures ($\widehat{PD}$) due the COVID-19 pandemic, relative to the benchmark year. While not explicitly modelled in our framework, these aggregate expenditures likely react to COVID lockdowns and other COVID shocks via income and precautionary savings channels. We can implicitly capture these effects by calibrating the change in aggregate expenditures using a measure that accounts for the effects of these channels. We therefore calibrate aggregate demand shocks using quarterly country GDP growth predictions, constructed by the IMF for the June 2020 World Economic Outlook Report.[28] These early forecasts account for the likely reaction of aggregate income to *all* COVID shocks.

Sectoral Productivity Shock: Many on-site workers in the benchmark (pre-COVID) year were forced to shift to remote work during the COVID-19 pandemic. The sectoral productivity shock ($\hat{A}_s$) captures possible declines in productivity due to this transition.

We assume sectoral productivity is a weighted average of the productivity of on-site and remote workers:

$$\begin{aligned} A_s &= A_s^{work}\theta_s + A_s^{home}(1-\theta_s) && \text{Before COVID,} \\ A_s' &= A_s^{work'}\theta_s' + A_s^{home'}(1-\theta_s') && \text{COVID-19,} \end{aligned} \tag{28}$$

where θ_s is the fraction of on-site workers, A_s^{work} is productivity of workers on-site and A_s^{home}

[28]It is worth nothing that the June 2020 WEO forecast may be impacted by some of the early fiscal policies implemented in our sample of countries. Typically, the WEO GDP growth forecast is submitted by individual country desks. The forecasts that are submitted include all the announced and implemented packages at the time that the forecasting round is closed. The June 2020 update was released on June 24. The forecasting round was likely finalized in early June, at the latest. An Italian package (for example) announced and voted in March 2020 would be factored into the June 2020 forecast, but one voted in June 2020 likely would not be.

is productivity of remote workers in each sector.

If we assume that A_s^{work} and A_s^{home} are constant (i.e. do do not change because of COVID), then we can express the sectoral productivity shock as:

$$\hat{A}_s = \frac{\theta'_s + \frac{A_s^{home}}{A_s^{work}}(1-\theta'_s)}{\theta_s + \frac{A_s^{home}}{A_s^{work}}(1-\theta_s)}. \tag{29}$$

We assume that firms in essential sectors are not forced to shift to remote work. Consequently, in essential sectors, $\theta'_s = \theta_s$ and $\hat{A}_s = 1$. Because firms in non-essential sectors can only employ remote workers during the lockdown period ($\theta'_s = 0$), Eq. (29) collapses to:

$$\hat{A}_s = \frac{\frac{A_s^{home}}{A_s^{work}}}{\theta_s + \frac{A_s^{home}}{A_s^{work}}(1-\theta_s)}. \tag{30}$$

To calibrate the sectoral productivity shock in non-essential sectors, we first use data from the 2018 American Community Survey (ACS) to calculate θ_s as the share of remote workers pre-COVID, by industry. Absent any good data on the relative productivity of on-site and remote workers, we opt to calibrate $A_s^{home}/A_s^{work} = 0.8$. This implies that $\hat{A}_s = 0.8$ (i.e. a 20% decline) is the maximum reduction in sectoral productivity, which would occur in a sector with no remote work before COVID and 100% remote work during the crisis.

5.3 Evaluating a Baseline COVID-19 Scenario

We first examine the vulnerability of countries, sectors, and firms to the COVID-19 crisis by evaluating a baseline scenario, absent government support. We model COVID-19 as a lockdown occurring for 8 weeks beginning in week 9 of 2020. During this lockdown, the sectoral labor supply ($\hat{x}_s$) and productivity ($\hat{A}_s$) and total demand ($\hat{d}_s = \tilde{\xi}_s^{\eta}\widehat{PD}$) shocks are active. After the lockdown ends, sectoral labor supply and productivity shocks return to benchmark year levels. Total demand continues to evolve throughout the year, with the aggregate demand component evolving according to IMF projections, and the sector-specific demand shock decaying according to an AR(1) process with quarterly persistence of 0.5. The evolution of total demand captures the subdued demand that persisted even after stay-at-home order were lifted because of continued uncertainty and fear of infection.

We use 2018 firm level Orbis data to measure benchmark (initial) firm sales, input cost shares, cash flow, cash balances, and financial expenses.[29] In the baseline scenario, we again

[29] In our baseline scenario we use 2018 data because it was the most recent and complete balance sheet data

make three assumptions, described in Section 2. First, cash flows are estimated weekly to reflect the evolution of COVID-19 shocks throughout 2020; but the liquidity criterion is only evaluated at the end of 2020 to capture that firms can smooth cash flow over the course of the year.[30] Second, firms are allowed to temporarily mothball. Third, because financial markets functioned well throughout 2020, we assume that firms have access to financing such that they may maintain their pre-existing debt levels and need only pay the interest due on this debt. We classify firms as failing if by the end of 2020, they have insufficient cash flow and cash balances to cover their financial expenses. That is, we have a single assessment period for whether a firm is illiquid which occurs at the end of 2020.[31]

We focus our analysis on understanding the effects COVID (and policy support) had on firm exit rates. In Appendix C we show that based on ex-post (though still estimated for most countries) data, it appears that entry rates *fell* during COVID suggesting that focusing on gross exit understates the decline in the number of operating firms during this period.

5.3.1 Estimating Aggregate SME Failure Rates

Table 3 reports our baseline, aggregate results. Column (1) reports the predicted 2020 failure rate in the absence of COVID-19 (non-COVID scenario) and serves as a useful benchmark. The non-COVID failure rate is calculated as a "typical year" scenario, as described in Section 5.1. Column (2) reports the end of 2020 estimated SME failure rate under the baseline COVID-19 scenario. Column (3) reports the difference between the two (Δ), and represents the excess SME failures in 2020. Throughout the remainder of the text, the excess failure rate is our preferred metric and is defined as the difference between a COVID-19 scenario and the non-COVID scenario. We find that the COVID-19 crisis results in a 6.01 percentage point excess SME failure rate.

Table 3: Aggregate SME Failure Rate

	Non-COVID	COVID	Δ
Average	9.53	15.55	6.01

Notes: Reports the estimated (1) non-COVID and (2) baseline COVID failure rates, and (3) the excess failure rate (Δ = baseline COVID - non-COVID). Failure rates are first calculated at the 1-digit NACE level and aggregated to the country level using 2018 sector GVA as weights. Failure rates are aggregated across countries using GVA as weights.

Our baseline results depend on a variety of assumptions regarding whether firms can ac-

available in June 2020.

[30] It is worth noting that over 90% of firms in most countries in our sample report their financial statements in the fourth quarter of the year. The two exceptions are Finland and France, where between 60 and 70% of firms report their financial statements in the fourth quarter.

[31] In a companion piece Gourinchas et al. (2021), we investigate the effects of COVID-19 and the wind-down of policy support on failures in 2021.

cess additional credit, mothball during 2020, or adjust prices. In Tables 4, 5 and 6 we show how these assumptions affect our estimates of excess failure rates in COVID, absent government support.[32]

First, Table 4 evaluates the effect of adjusting the number of assessment periods on excess failure rates. In our baseline (repeated in column (1) of the table), we allow firms to run temporary cash deficits until the end of 2020. Columns (2) through (5), respectively, show the effects of having assessment periods twice a year, quarterly, monthly and weekly.[33] As the assessment period becomes more frequent, failure rates rise because fewer firms are able to maintain positive cash balances at the end of each assessment period. With weekly assessment periods—equivalent to assuming that firms having zero access to additional credit, while still being able to rollover pre-existing debt—we estimate an excess failure rate of 8.09 percentage points. The higher excess failure rate, relative to our baseline, points to credit lines having a potentially meaningful effects on firms' ability to survive the COVID-19 shocks.

Table 4: Alternative Assessment Periods

	(1) Baseline	(2) Bi-annual	(3) Quarterly	(4) Monthly	(5) Weekly
Average	6.01	7.24	7.68	8.03	8.09

Notes: Reports the excess failure rates (COVID - non-COVID) under the following scenarios that differ in the assessment period: Baseline, which reflects an annual assessment period (col. 1); bi-annual (col. 2), quarterly (col. 3), monthly, (col. 4), and weekly (col. 5). Failure rates are first calculated at the 1-digit NACE level and aggregated to the country level using 2018 sector GVA as weights. Failure rates are aggregated across countries using GVA as weights.

Next, Table 5 shows the effects of alternative assumptions about firms' ability to access new credit. In order to implement these scenarios, we impose one of Eq. (22) and Eq. (23) on new borrowing ($B'_{is,t} - B_{is,t}$) in our cash flow equation (Eq. (21)). Specifically, we consider:[34]

$$B'_{is,t} \leq \alpha_K \times \text{Tangible Fixed Assets}_{is} \qquad \alpha_K \in \{0.25, 0.5, 0.67\} \tag{31}$$

$$B'_{is,t} \leq \alpha_E \times \text{EBITDA}_{is} \qquad \alpha_E \in \{2.5, 3.0, 3.5\} \tag{32}$$

[32] In all tables we report *excess* failure rates – that is the failure rate in COVID in excess of what we would expect in 2020 had COVID not occurred. As we change each modelling assumption, we also implement the same change to our non-COVID scenario.

[33] One small technical point relevant to these scenarios is that they are partial equilibrium in the sense that expenditure at firms that have exited is not redistributed to the remaining surviving firms. The reason for this is to showcase the direct effect these financing assumptions can have on firm health. However, once there are periods in the simulation *after* some firms have failed, a proper analysis should incorporate this demand redistribution channel. We repeat Table 4 in Appendix F with this adjustment for interested readers.

[34] When imposing this constraint we never lower a firms' borrowing below the pre-shock level. As such $B'_{is,t} \geq B_{is,t}$.

Eq. (31) represents an asset based borrowing constraint that limits firms' total borrowing to a fraction (α_K) of their benchmark period tangible fixed assets. Eq. (32) represents an earnings based borrowing constraint that allows firms to borrow up to some fraction (α_E) of their benchmark period EBITDA. This latter constraint is similar in spirit to the credit line approach in our baseline scenario. Both are earnings based constraints, except that in Eq. (32) firms face a borrowing constraint based on pre-shock earnings, whereas in our baseline scenario, firms face a borrowing constraint based on their post-shock earnings. When implementing these pre-shock scenarios we assess whether firms fail at a weekly frequency, which means that when these borrowing constraints bind firms fail as soon as their cash flow is negative).[35]

Table 5 shows that these borrowing constraints can have a meaningful effect on excess failure rates. Column (1) repeats our baseline scenario. Columns (2) through (4) show scenarios with borrowing constraints based on tangible fixed assets where α_K is set to 25%, 50% and 67%, respectively. In all cases, excess failure rates are similar to our baseline with excess failure rates slightly below our baseline scenario when firms can borrow up to 67% of their tangible fixed assets. We see large divergences when we allow firms to borrow based on their *pre-shock* EBITDA. Excess failure rates fall considerably below the baseline level. However, it is worth nothing that it is likely that an earnings-based borrowing constraint of 3 or higher on *pre-COVID* EBITDA is unlikely in a scenario like COVID where the earnings of many firms collapsed.

Table 5: Alternative Formulations of the Borrowing Constraints

		Tangible Fixed Assets			EBITDA		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Baseline	25%	50%	67%	2.5	3.0	3.5
Average	6.01	6.99	6.29	5.83	4.20	3.84	3.49

Notes: Reports the excess failure rates (COVID - non-COVID) under the following scenarios: Baseline (col. 1); borrowing constraint based on tangible fixed assets, such that (existing borrowing + new borrowing)/tangible fixed assets $\leq \theta$. θ is set to 25% (col. 2), 50% (col. 3), and 67% (col. 4); and borrowing constraint based on debt/EBITDA ratio, such that (existing borrowing + new borrowing)/EBITA $\leq \gamma$. γ is set to 2.5 (col. 5), 3.0 (col. 6), and 3.5 (col. 7). Failure rates are first calculated at the 1-digit NACE level and aggregated to the country level using 2018 sector GVA as weights. Failure rates are aggregated across countries using GVA as weights.

Table 6 reports the effects of two additional extensions. First, we no longer allow firms to "mothballing", or temporarily close (col. 2). Relative to our baseline (col. 1), excess failure rates rise by less than one percentage point when firms cannot mothball. Second, we allow firms to adjust prices in response to COVID shocks (cols. 3 to 6). Because the effect of flexible prices depends on the elasticity of demand (η) over sectors, we consider scenarios where $\eta = 0.2$, $\eta = 1$ and $\eta = 2$ (cols. 3-5 respectively). Thus we cover cases where there is a

[35] As discussed in the previous footnote, when there are multiple assessment periods demand from failing firms should be redistributed to the surviving firms. We implement this reallocation in Table 5.

high degree of complementarity between types of goods and services up to a high degree of substitutability. In all cases, excess failure rates *rise* by at least 3 percentage points (50%) relative to our baseline. Excess failure rates rise because under price flexibility, firms facing negative demand shocks face lower costs as they scale back production (due to our assumption of diminishing returns to scale). When prices are fixed, this automatically raises these firms per-unit markups. Under price flexibility and CES demand, firms seek to maintain the same markup as pre-COVID leading to lower prices and lower per-unit markups than the fixed price case. While this is individually rational for a single firm as they can gain market share by lowering their prices, if most firms are lowering their prices, then the average firm can end up in a situation where, relative to the fixed price scenario, they have lower per-unit markups but similar revenues. Thus our flexible price equilibrium delivers lower profits for the average firm and therefore raises failure rates. This mechanism operates independently of the elasticity of demand (η) and in all cases failure rates in our flexible price scenarios are above our fixed price benchmark.

There are situations in which firms can benefit from being able to adjust prices—for instance, when facing large cost pressures, firms can preserve cash flow by passing these costs onto consumers. In addition, when elasticities of demand are above 1, firms in sectors with the worst demand shocks can gain market share as they cut their prices. This leads these firms to have smaller falls in their revenues which helps preserve profits and cash flow. However in our simulations, these situations have considerably weaker effects on firm health than the cost pressures imposed on firms engaged in price competition in low demand sectors. Appendix E provides a more detailed discussion of these different situations and how they contribute to an overall rise in excess failure rates.

Finally, Appendix G re-estimates our baseline scenario, allowing for input-output (I-O) linkages across sectors. The extension changes the sectoral pattern of excess failure rates considerably, but the aggregate excess failure rates are similar to those estimated under flexible prices.[36]

[36] Note that if we impose fixed prices, then I-O linkages only have effects on the level of demand facing firms. To have supply bottleneck effects on firms through materials prices, we need to allow firms to adjust prices.

Table 6: Excess Failure Rates (Δ) under Extensions

	Baseline	No Mothbaling	Flexible Prices		
			($\eta = 0.2$)	($\eta = 1$)	($\eta = 2$)
	(1)	(2)	(3)	(4)	(5)
All	6.01	6.69	13.48	15.63	12.02

Notes: Reports the excess failure rates (Δ = baseline COVID - non-COVID) under—(1) baseline scenario: annual liquidity criterion evaluation and firms are allowed to mothball; (2) annual liquidity criterion and no mothballing; (3) flexible prices with an elasticity of demand across sectors (η) of 0.2; (4) flexible prices with an elasticity of demand across sectors (η) of 1; and (5) flexible prices with a price elasticity of demand of 2. Failure rates are first calculated at the 1-digit NACE level and aggregated to the country level using 2018 sector GVA as weights. Failure rates are aggregated across countries using GVA as weights.

5.3.2 Exploring Sources of Sectoral and Country Heterogeneity

Considerable heterogeneity underlies our average estimate of a 6.01 percentage point excess SME failure rate—the excess failures were much higher in some country-sectors and much lower in others. Because our framework estimates failures at the firm level, we can study how individual firms with different initial financial conditions respond to shocks. This allows us to evaluate sources of heterogeneity in sector and country outcomes and to compare characteristics of failing firms in COVID to those that fail in a typical year.

Sectoral Exposure to Shocks: Table 7 confirms that there is considerable variation across sectors in excess failure rates under COVID-19. Columns (1) and (2) report the non-COVID and baseline COVID-19 SME failure rates, respectively. Column (3) reports the excess failure rate (Δ). Given their customer orientation and limited scope for remote work, some service sectors, such as Accommodation & Food Service or Arts, Entertainment & Recreation, experience excess failure rates exceeding 10 percentage points. In stark contrast, majority-essential 1-digit sectors (henceforth referred to as "essential sectors" and highlighted in gray), including Construction and Transport & Storage, that face small sectoral supply shocks and higher sector-specific demand, experience less than 3 percentage point excess SME failure rates.[37] Finally, sectors with fewer essential workers, but relatively low total demand shocks and/or high scope for remote work (Professional, Scientific & Technical Services) are moderately affected, experiencing excess failure rates between 5 and 10 percentage points.

To better understand which COVID-19 shocks drive the observed cross-sector variation, Table 8 evaluates changes in excess failure rates under five alternative scenarios that differ in the composition of COVID-19 shocks.[38] The first column only includes the aggregate demand shock ($\widehat{PD}$). The second column includes both sectoral and aggregate demand shocks (or total demand shock, $\widehat{PD}\tilde{\xi}_s^{\eta}$). The third includes both aggregate demand and sectoral labor supply

[37] Note that in some essential sectors, total demand can *rise* in COVID-19 and this can lead to lower failure rates than in a normal year—see Water & Waste.

[38] Additional shock combinations are presented in Tables 25, 26, 27, and 28 in Appendix H.

Table 7: Sector SME Failure Rates

	Non-COVID	COVID	Δ
Agriculture	8.65	9.64	0.98
Mining	9.59	14.72	5.13
Manufacturing	8.46	10.38	1.92
Electric, Gas & Air Con	9.21	9.33	0.12
Water & Waste	7.80	7.33	-0.47
Construction	7.52	7.62	0.10
Wholesale & Retail	8.74	17.62	8.87
Transport & Storage	8.63	10.20	1.56
Accom. & Food Service	12.63	25.94	13.31
Info. & Comms	10.12	13.80	3.68
Real Estate	11.43	17.41	5.97
Prof., Sci., & Technical	10.54	17.33	6.79
Administration	8.02	19.05	11.02
Education	11.06	30.55	19.49
Health & Social Work	8.32	10.91	2.59
Arts, Ent., & Recreation	12.14	31.51	19.37
Other Services	13.32	28.20	14.88

Notes: Reports the estimated (1) non-COVID and (2) baseline COVID failure rates, and (3) the excess failure rate (Δ = baseline COVID - non-COVID). Sector failure rates are first calculated at the 1-digit NACE level for each country, and then aggregated across countries using (country x sector) GVA as weights. 1-digit sectors where the majority of 4-digit sectors are classified as essential are highlighted in gray.

shocks $(\widehat{PD}, \hat{x}_s)$. The fourth includes total demand and sectoral labor supply shocks $(\widehat{PD}\tilde{\xi}_s^{\eta}, \hat{x}_s)$. The last is our baseline, which adds sectoral productivity shocks to the fourth column.

Column (1) shows that when only the aggregate demand shock is included, excess failure rates range from 0.05 percentage points in Mining to 7.23 percentage points in Transportation & Storage. Because all sectors in a country face identical aggregate demand shocks, this heterogeneity must stem from differences in firm financial health across sectors. By this metric, Transport & Storage is ex-ante one of the most financially vulnerable sectors. This ex-ante vulnerability can arise from, for example, low cash balances and/or high debt levels, which increase the likelihood that declines in cash flow lead to liquidity shortages.

The addition of sector-specific demand shocks to the aggregate demand shock (col. 2) either exacerbates or mitigates underlying sectoral vulnerability, thus resulting in higher excess failure rates in some sectors and lower excess failure rates in others. In an already vulnerable sector, like Administration, even a modest negative sector-specific demand shock leads to a large rise in excess failure rates. Meanwhile, according to column (1) Transport & Storage is the most ex-ante vulnerable sector and Arts, Entertainment & Recreation among the least. Yet, because sector specific demand falls most in customer-oriented service sectors, like Art, Entertainment & Recreation, and increases in essential sectors, like Transport & Storage, excess SME failure rates in column (2) rise in Arts, Entertainment, & Recreation far above those in Transport & Storage.

Adding the sectoral labor supply shock to the *aggregate* demand shock (col. 3) heavily impacts non-essential, labor-intensive sectors that cannot easily transition to remote work, such as Accommodation & Food Service. The pronounced rise in excess SME failure rates in these sectors occurs because a small aggregate demand shock, relative to a more severe labor supply shock, leads to a high fraction of firms becoming labor constrained. For these firms to meet demand, they must make a costly substitution away from labor, which deteriorates their cash flow and leads to a liquidity shortage.[39] Meanwhile, labor-intensive sectors with higher capacity for remote work, such as Information & Communications, experience a smaller rise in excess failure rates. Sectors composed of essential sub-sectors, such as Construction and Transport & Storage, are exposed to small labor supply shocks and therefore experience only a small rise in excess failure rates.

The addition of sector-specific demand shocks to aggregate demand and sectoral labor supply shocks (col. 4) is informative about which shock—sectoral labor supply or sector-specific demand—is more binding for sectors. In some sectors, like Accommodation & Food Service and Mining, the addition of the sector-specific demand shock does not raise excess failure rates much above those in in column 3, pointing to the importance of sectoral labor supply shocks. In contrast, the sector-specific demand shock appears more important than the sectoral labor supply shock in a sector like Arts, Entertainment, & Recreation. Comparing columns (4) to (5) shows the effects of the productivity shock on sectoral excess failure rates, which in this case is modest.

Table 8: Excess Failure Rate (Δ) Comparison (Alternative Shock Combinations)

	$\widehat{PD}$	$(\widehat{PD\tilde{\xi}_s})$	$\widehat{PD}+\hat{x}_s$	$(\widehat{PD\tilde{\xi}_s}),\hat{x}_s$	Baseline
Agriculture	0.73	0.38	1.26	0.97	0.98
Mining	0.05	0.41	4.12	4.84	5.13
Manufacturing	1.04	0.75	2.13	1.97	1.92
Electric, Gas & Air Con	1.12	0.07	1.12	0.07	0.12
Water & Waste	3.60	0.49	3.60	0.49	-0.47
Construction	1.81	-0.33	1.85	-0.34	0.10
Wholesale & Retail	2.18	8.76	2.86	8.56	8.87
Transport & Storage	7.23	1.21	7.24	1.22	1.56
Accom. & Food Service	0.09	7.85	10.27	11.79	13.31
Info. & Comms	1.77	3.15	1.92	3.15	3.68
Real Estate	1.60	6.04	0.97	6.03	5.97
Prof., Sci., & Technical	3.40	6.80	3.14	6.71	6.79
Administration	4.28	9.35	4.46	9.35	11.02
Education	2.35	19.01	12.73	19.01	19.49
Health & Social Work	2.11	2.50	3.48	2.50	2.59
Arts, Ent., & Recreation	1.88	18.58	10.60	18.82	19.37
Other Services	0.07	14.56	7.35	14.87	14.88
Average	2.16	5.36	3.71	5.72	6.01

Notes: Reports the excess failure rate (COVID - non-COVID) under five scenarios—(1) aggregate demand shock only ($\widehat{PD}$); (2) aggregate demand and sector-specific demand shocks ($\widehat{PD\tilde{\xi}^{\eta}_s}$); (3) aggregate demand and sectoral supply shocks ($\widehat{PD},\hat{x}_s$); (4) total demand and supply shocks ($\widehat{PD\tilde{\xi}^{\eta}_s},\hat{x}_s$) and; (5) the baseline ($\widehat{PD\tilde{\xi}^{\eta}_s},\hat{x}_s,\hat{A}_s$). Sector excess failure rates (Δ) are first calculated at the 1-digit NACE level for each country, and then aggregated across countries using (country x sector) GVA as weights. The last row is the sector GVA weighted average. 1-digit sectors where the majority of 4-digit sectors are classified as essential are highlighted in gray.

Country-Specific Factors: Other than the evolution of $\widehat{PD}$, our baseline COVID-19 scenario

[39] While the worst affected can mothball during the lockdown, they still face cash flow reductions while closed due to fixed costs.

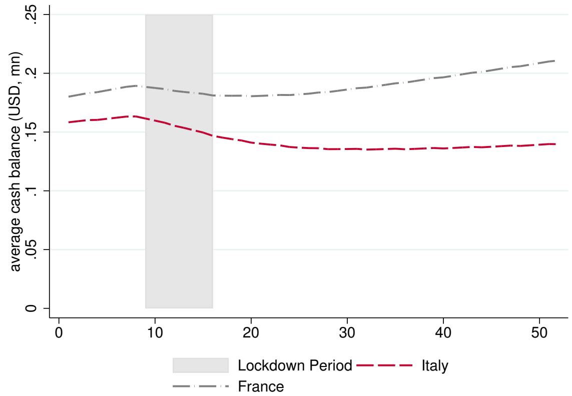
Table 9: Country-Level SME Failure Rates

	Non-COVID	COVID	Δ
Czech Republic	7.36	9.92	2.56
Finland	10.17	14.34	4.18
France	10.15	15.42	5.27
Hungary	8.86	11.63	2.77
Italy	9.24	19.54	10.30
Poland	11.88	17.39	5.50
Portugal	12.15	16.17	4.02
Romania	11.90	14.28	2.37
Slovak Republic	9.27	12.29	3.02
Slovenia	6.36	9.34	2.98
Spain	7.51	11.26	3.75

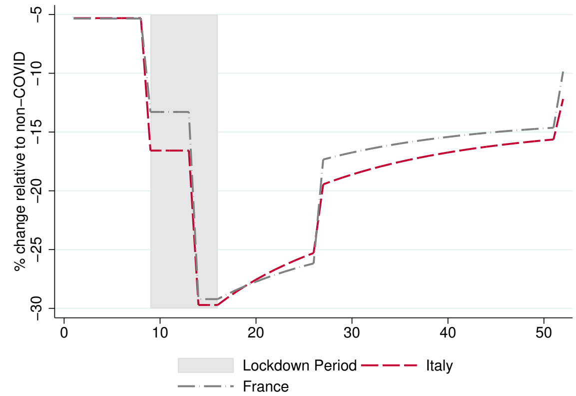
Notes: Reports the estimated (1) non-COVID and (2) baseline COVID failure rates, and (3) the excess failure rate (Δ = baseline COVID - non-COVID). Country level results represent the weighted average of 1-digit NACE failure rates, where weights are given by sector GVA.

features identical shocks for all firms that operate in the same sector, irrespective of country. Nonetheless, as Table 9 documents, there is considerable cross-country heterogeneity in excess SME failure rates (Δ, col. 3), ranging from 2.37 percentage points in Romania to 10.30 percentage points in Italy.

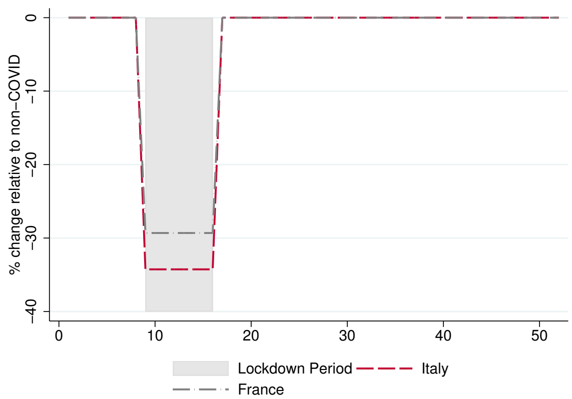
To better understand the sources of heterogeneity, in Fig. 5 we compare France and Italy. Under our baseline scenario, Italy's excess SME failure rate is 5.03 percentage points higher than France's. Fig. 5 makes clear the importance of both industrial composition and overall firm financial health in explaining the differential impact of COVID-19 across these two countries. The figure depicts the weekly evolution of (a) average firm cash balances divided by initial total assets, (b) total demand shocks, (c) sectoral supply shocks and (d) fraction of firms that are labor constrained.



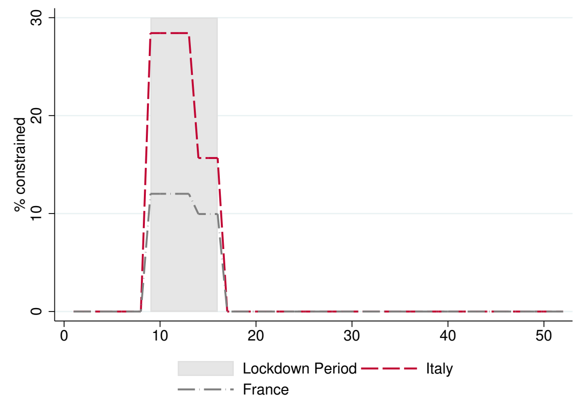
(a) Cash Balance/Initial Total Assets



(b) Total Demand Shock



(c) Sectoral Labor Supply Shock



(d) Fraction of Firms Constrained

Figure 5: Weekly Evolution of Variables of Interest (Country)

Notes: Figures show the weekly evolution of (a) average firm cash balance divided by initial total assets, (b) total demand shock (interaction between sector-specific demand and aggregate demand shock), (c) sectoral labor supply shock, and (d) fraction of firms constrained. In each week, country-level variables represent the weighted average of 1-digit NACE variables, where weights are given by 2018 sector GVA.

While firms in a given sector face the same sectoral shocks regardless of the country they are in, the country *averages* of these shocks can vary based on differences in industrial composition. Total demand evolves similarly in France and Italy, as does the sectoral labor supply shock. However, because more Italian firms are in sectors facing relatively modest sector-specific demand shocks but stringent workplace restrictions, a higher fraction of firms become labor constrained. This means that Italian firms face higher costs during the lockdown than French firms. The largest difference between the two countries is firms' initial cash-to-assets ratio. Italian firms begin COVID with less cash, relative to their total assets, than French firms, which makes them more likely to fail under COVID.

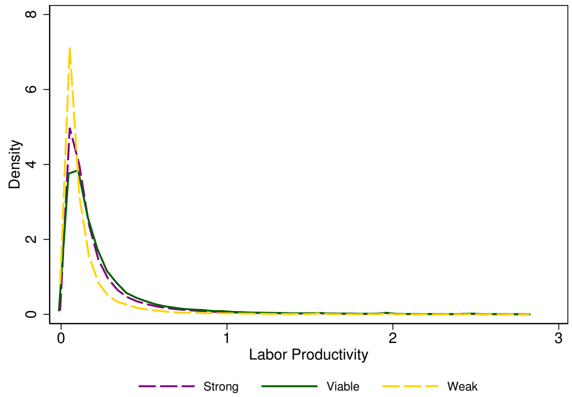
5.3.3 Examining Firm Level Heterogeneity

In Section 4, we showed that in typical years, failing firms have lower labor productivity, profitability, revenue growth, and cash balances than surviving firms. In our baseline COVID scenario, many more firms fail than in a typical year. These high excess SME failure rates raise the question of whether the additional failing firms continue to differ considerably from surviving firms.

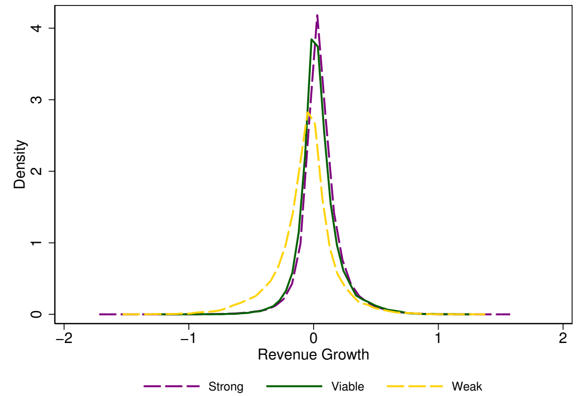
To shed light on this question, we divide firms into three groups. The first group is "strong" firms that remain liquid through the end of 2020 in our baseline COVID-19 scenario. We then split firms that fail in the baseline COVID scenario into two subgroups—"weak" firms that would fail even if COVID never occurred (i.e. under the non-COVID scenario) and "viable" firms that survive the non-COVID scenario but fail in the baseline COVID scenario. Note that these firm groups are defined based on their survival under the *baseline* COVID scenario, relative to the non-COVID scenario. As such, their composition is invariant to the policy counterfactuals that we evaluate in the next section.

Fig. 6 and Table 10 compare the three firm groups. Panel (a) of Fig. 6 shows that, surprisingly, "viable" firms have higher labor productivity than *both* "strong" and "weak" firms. Panel (b) reports the average of past revenue growth, and again "viable" and "strong" firms look similar. Panels (c) and (d) show the cash-to-assets ratio and short-term leverage distributions, respectively. Here, "viable" firms look more similar to "weak" firms—they have lower cash balances and higher short-term leverage than "strong" firms. Table 10 further shows that, like "strong" firms, "viable" firms are profitable; but, like "weak" firms, are smaller and younger than "strong" firms.

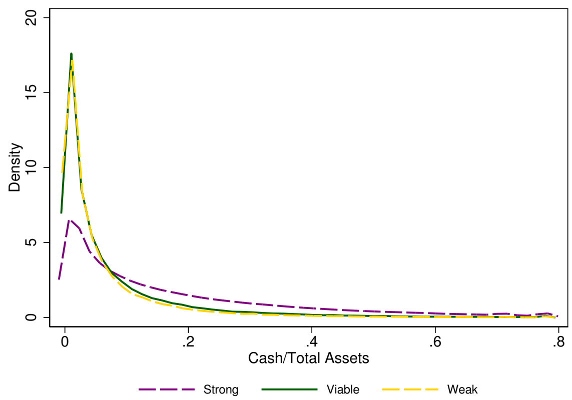
Taken together, it appears that "viable" firms are likely to fail in the baseline COVID scenario due to their low cash buffers and high financial obligations. Given their strong labor productivity, profitability, and past growth, there is potentially a case to be made for preventing the failure of these firms.



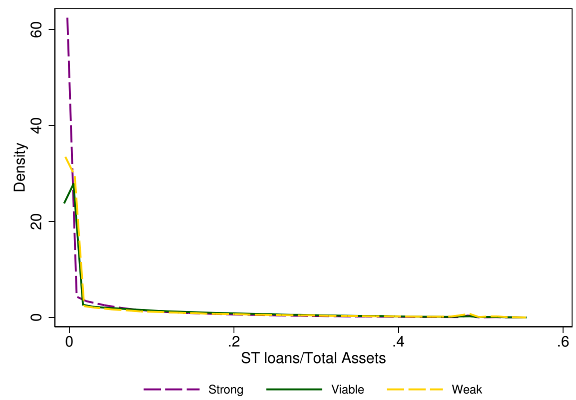
(a) Log Labor Productivity



(b) Revenue Growth Rate (3-year Average, %)



(c) Cash/Total Assets (%)



(d) Short-term Leverage (%)

Figure 6: Distributions of Variables by Firm Types in Baseline COVID Scenario

Notes: Depicted are the distributions of (a) log labor productivity (defined as sales per worker), (b) revenue growth rate in percent, (c) beginning of period cash-to-total assets ratio and (d) short-term leverage (defined as short-term loans/initial total assets) of firms that we estimate fall into one of three groups: "strong" that survive both under the non-COVID and baseline COVID scenarios; "viable" that fail under baseline COVID only; and "weak" that fail under both non-COVID and baseline COVID.

Table 10: Summary Statistics (Median): Strong, Viable, and Weak Firms

	Strong	Viable	Weak
Cash/Total Assets	10.85	2.92	2.08
EBITDA/Total Assets	9.73	5.75	-12.93
Employment Growth	0.72	0.49	-0.61
Firm Age	15.78	12.67	12.65
Labor Productivity	0.16	0.17	0.11
Number of Employees	12.47	9.92	9.90
Revenue (Millions USD)	2.23	1.83	1.30
Revenue Growth	3.57	2.18	-4.10
Short-term Loans/Total Assets	1.02	1.77	1.02

Notes: Table reports the median of variables of interest, separately for firms that we estimate to be (1) "strong" that survive both under the non-COVID and baseline COVID scenarios, (2) "viable" that fail under baseline COVID only and (3) "weak" that fail under both non-COVID and baseline COVID.

5.4 Evaluating Policy Counterfactuals

Our baseline scenario indicates which countries, sectors and types of firms are particularly vulnerable to the COVID-19 shocks in the absence of government support. We now consider several policy counterfactuals to highlight how our framework can be used to study the cost and impact of policy alternatives, as well as gauge the types of firms that these policies save.

We implement policy support in 2020 as a lump-sum cash injection to firms:

$$CF'_{is} = CF_{is} + (\pi'_{is} - \pi_{is}) + G'_{is}$$

where G'_{is} represents funds coming from policy support. We allow for policy support to be a function of firm balance sheet variables *in the benchmark year* to avoid accounting for the impact policy may have on firm choices during COVID. Notice that the method by which resources are transferred to firms (e.g. tax rebates or government guaranteed loans) is irrelevant to firms in 2020, the period which our exercise covers. To avoid failure, all that matters to a firm is the injection of additional resources (or reduction in expenses due) it receives (or owes). The form of the policy support (grant vs. loan) will however affect its net cost to government.

We make several assumptions when implementing the policy counterfactuals. First, we assume that aggregate demand evolves in exactly the same manner as in our baseline COVID scenario. To the extent that saving some firms and preserving some jobs may raise aggregate demand, the numbers presented here likely understate the overall effect of policy support. Further, because we assume perfectly rigid prices and wages, we do not capture worker reallocation effects or the possible impact of policy support on such reallocation. Finally, an assumption underlying our discussion of policy performance during COVID is that governments prefer channeling funds to firms that fail specifically due to the COVID-19 shock ("viable" firms) than to firms that either do not need the support ("strong" firms) or would have failed even if COVID-19 had not occurred ("weak" firms).[40]

5.4.1 Evaluating Fiscal Policy Scenarios

For each policy we consider, Table 11 shows the costs and possible benefits of saving SMEs. The first column shows the percent of all firms saved by each policy, which we define as the difference between the excess failure rate in the baseline COVID-19 scenario and the excess failure rate when each policy is implemented. The second column shows jobs preserved at firms that no-longer exit under each policy, as a fraction of total employment. The third column reports

[40] This is not an innocuous assumption. Ideally, the firms targeted by policy would be those that are temporarily illiquid in COVID, but have a positive expected present value of profits. Some such firms could easily be in both our "weak" and "viable" firm groupings, but we expect most such firms will be in our "viable" firms group.

the amount of wages payments preserved, which we define as the total labor compensation that is preserved under each policy, as a share of gross value added (GVA). These numbers take into account that saved firms may choose to operate at lower scale—employing fewer workers and paying less in labor compensation—than in pre-COVID.[41] The fourth column reports firm net worth preserved (defined as book value of assets less liabilities), and the fifth column reports the funds disbursed to firms by each policy, expressed as a fraction of GVA.[42]

To benchmark the performance of policies implemented in practice, the first row of Table 11 considers a hypothetical policy that bails out every firm that fails specifically because of the COVID-19 crisis (i.e. "viable" firms). Under this policy, each "viable" firm receives the minimum amount required to leave it with a zero cash balance at the end of 2020. While this policy is feasible in our framework, the identity of "viable" firms and their cash deficits are not observable in practice. Nonetheless, we find this policy to be a useful benchmark because it approximates the level of resources that would be required if governments wanted to *fully* mitigate the impact of COVID-19 on "viable" firm failures.

Table 11: The Impact and Costs of Various Policy Scenarios

	(1)	(2)	(3)	(4)	(5)
	Firms Saved (% Firms)	Jobs Saved (% Employed)	Wages Saved (% GVA)	Net Worth Preserved (% GVA)	Funds Disbursed (% GVA)
Benchmark Policy	7.29	3.10	1.44	15.49	0.77
Financial Expenses Waived	1.67	0.66	0.33	7.43	1.43
Tax Waiver	2.21	0.80	0.28	3.05	1.61
Rent Waiver	3.97	2.22	0.97	5.24	3.05
Cash Grant	4.74	2.63	1.14	4.96	2.63
Pandemic Loans	7.85	4.02	1.80	10.44	6.43

Notes: Because Orbis does not cover the universe of firms, we calculate aggregate costs by scaling the total costs in Orbis by the inverse of the coverage ratio of Orbis (based on 2018 value added for policy costs, total remuneration for wages saved, and employment at the 1-digit NACE level). The numbers presented here are GDP-weighted averages across countries.

* Unlike the other policies, the funds disbursed under the pandemic loan policy do not equal the fiscal cost, which depends on the rate of repayment and the distribution of losses between the government and the banking sector.

Our benchmark policy illustrates that, provided sufficient information, the fiscal cost of saving "viable" SMEs could be modest. With an overall disbursements of 0.77% of GVA, the benchmark policy preserves 7.29% of firms, 15.5% of GVA in firm net worth, up to 3.10% of

[41]These jobs and wages saved numbers pertain specifically to jobs and wages "preserved" for 2020 because firm failures were prevented. Wages preserved are calculated as the wages that would be paid over all of 2020 by firms that we estimate failed in 2020. When these firms cut production in 2020 in response to COVID shocks, our wages preserved calculations reflect this. Jobs preserved are calculated as the end of 2020 level of employment each failing firm would choose under the assumption that they did not fail in 2020. These calculations may understate the long-run jobs and wages saved if saved firms eventually return to their previous scale as they recovered from the COVID-19 shock or if policy support created jobs.

[42]We consider additional measures of firm value in Appendix J.

jobs, and 1.44% of GVA in wages in 2020.[43,44] Moreover, each dollar disbursed by this policy generates up to \$1.87 in direct aggregate demand (1.44/0.77) in the form of wages saved. We call this ratio the *fiscal-bankruptcy multiplier*. This multiplier is not a traditional Keynesian multiplier; it reflects that productive, high growth businesses may be shut down as a consequence of the pandemic, and that fiscal resources deployed to preserve "viable" businesses help increase overall output and employment.[45]

The next five rows of Table 11 consider a set of alternative policies that better reflect the policy responses implemented by countries.[46] Rather than focus on the policies of any particular country, we focus on policy interventions that together span most types of policies implemented by governments. Policy responses have varied considerably across countries but have tended to take the form of cheaper debt refinancing, loan guarantees, expense rebates, and size-based grants.

The first set of policies rebate to firms their financial expenses (row 2 of Table 11), taxes (row 3) or rent (row 4) at the beginning of lockdown through the end of 2020.[47] The financial expenses and tax rebates have in common that they can be implemented at moderate cost, but have modest impacts. For example, under the financial expenses rebate, 1.67% of firms are saved, at a cost of 1.43% of GVA. The fiscal bankruptcy multiplier is low at 0.33/1.43 = 0.23. Meanwhile, waving rents is a bit costlier, at 3.42% of GVA and saves more firms, 4.14%.[48] Yet, the fiscal-bankruptcy multiplier remains low at 1.00/3.42 = 0.29.

The last two policies considered are injections of new funds rather than rebates. The first of these is a cash grant that disburses to firms their average 2018 weekly wage bill during the 8 weeks of lockdown.[49] Importantly, because the payments are lump-sum, assessed on the basis

[43] 7.29% of firms are viable despite excess failure rates being only 6.01% (Table 3). This difference is accounted for by the existence of 1.27% of firms that fail in our non-COVID scenario yet *survive* COVID because some (essential) sectors faced higher demand during COVID. The positive demand shocks helped save these firms from otherwise failing in 2020. These firms are classified as strong firms and offset some of the rise in excess failure rates.

[44] Note that Orbis does not cover the full universe of firms. To compute columns (2), (3) and (5) in Table 11, we calculate sectoral coverage rates by comparing 1-digit sectoral Orbis employment and labor costs to the equivalent OECD data for each country. We then scale by the inverse of the coverage ratio to get representative numbers for each country by sector pair.

[45] Traditional fiscal multipliers would differ—one dollar in fiscal resources used to preserve viable businesses may increase overall output by more (or less) than 1.44 dollars. We ignore these general equilibrium considerations in this paper and focus on the first-round effects of the fiscal interventions.

[46] According to OECD (2020) tax deferrals have been one of the most common policy support measures used by OECD governments and 22 OECD countries have implemented some form of rent deferral or waiver scheme. Cash grants and government guaranteed loans are also widely used. See ECB Economic Bulletin 6/2020 Focus

[47] Note that the financial expenses rebate is an extreme version of policies that guarantee existing firm loans or refinance them at lower interest rates.

[48] Orbis does not include any information on firm rents. Therefore, we estimate firm rent expenses by assuming that the ratio of rent to cost-of-goods-sold is constant within 1-digit sectors and use data from Compustat to calculate these ratios.

[49] This grant therefore equals 8/52=15.4% of the 2018 wage bill of the firm. Cash transfers of this form are discussed in an early policy note from April 2020, Drechsel and Kalemli-Ozcan (2020).

of the wage bill in the benchmark year, they do not affect the current cost of labor or firms' employment decisions. These cash grants have a much larger impact than the rebate policies on business failures, jobs and wages preserved, though generally at a higher cost. The grant preserves 4.74% of firms, up to 2.63% of jobs and 1.14% of GVA in wages, but at an overall cost of 2.63% of GVA.[50] The fiscal-bankruptcy multiplier is 0.43—each dollar of fiscal resources preserves 0.43 cents in direct aggregate demand.

The final policy we consider is a program of public loan guarantees for SMEs (e.g. pandemic loans), broadly similar to those implemented by several Euro-area countries.[51] Because most of the countries we focus on belong to the Euro-area, this policy is especially relevant. To remain consistent with how the policy was designed in Europe, we assume that zero interest and principal is due in 2020. Consequently, from the perspective of 2020 outcomes, the relevant aspects of the loan guarantees is the new injection of funds that help some SMEs survive the year. Other than affecting the policy's net cost, repayment terms and interest beyond 2020 have no effect on our analysis.[52]

This policy is the most generous, providing 6.43% of GVA in funding to SMEs.[53] It has a dramatic impact on failure rates, bringing them *below* their pre-COVID levels and preserving up to 4.02% of jobs.[54] At first glance, the fiscal bankruptcy multiplier, in terms of wages saved relative to funds disbursed, appears low at 1.80/6.43=0.28. The policy preserves 10.44% of firm net worth as a percent of GVA. However, as we will discuss later, because this policy is a loan, the fiscal bankruptcy multiplier once repayment is accounted for could easily be much higher.

5.4.2 Evaluating which Firms Get Saved

Our analysis shows that real world policies, including cash grants and pandemic loans, can be effective at preserving firms; but at costs that far exceed those required under our targeted benchmark policy. It remains to be seen which types of firms benefited most from these real world policies, both in terms of firms saved and money disbursed.

Table 12 decomposes the effects of a subset of policies on "strong", "weak" "viable" firms.

[50] Several sectors (e.g. the financial sector and the government sector) are not included in our analysis, which may help explain why the overall policy costs of this cash grant appear small.

[51] Under the terms of this program, firms are eligible to borrow up to the larger of 25% of their 2018 revenues, or twice their 2018 wage-bill, during each week of lockdown. They are not required to pay interest or repay any principal in 2020. See ECB Economic Bulletin 6/2020 Focus for details.

[52] Our companion paper, Gourinchas et al. (2021) explores the implications of repayment of this program on firm failures in 2021.

[53] This amount represents funds disbursed by the banking sector and not a policy cost. The policy cost will depend on the repayment rate and the distribution of losses between the government and banking sector.

[54] We assume funds are directly channelled from banks to firms, whereas in real-life these type of programs suffered several setbacks and delays due to frictions in banking intermediation.

We focus our attention on the cash grants and pandemic loan policies, and include our benchmark policy for comparison. Column (1) of Table 12 pertains to "strong" firms, columns (2) and (3) to "weak" firms, and columns (4) and (5) to "viable" firms. Columns (2) and (4) show the failure rates under each policy for the "weak" and "viable" firms.[55] For instance, under our benchmark policy, all "weak" firms fail because they do not receive any support, while the failure rate of "viable" firms falls to zero.[56] Columns (1), (3) and (5) show the funds disbursed to each group and column (6) the total amounts disbursed, all as a percent of GVA.

Table 12 highlights two features of the cash grant and pandemic loan policies. First, despite concerns that policies would primarily benefit "weak" firms, we find that the majority of firms saved are "viable". The pandemic loan policy (cash grant) policy preserves 60 (38)% of all "viable" firms, which account for 59 (55)% of all preserved firms.[57] The pandemic loan (cash grant) policy does also save 42% (24%) of all "weak" firms, which account for the remaining 45% (41%) of saved firms. Table 13 further shows that approximately 58% of the jobs preserved (1.53/2.63) and wages preserved (0.78/1.35) from the cash grants can be attributed to retaining workers at "viable" firms. The same figures for the pandemic loan policy are 55 and 54%, respectively.

Second, as shown in Table 12, despite concerns that most resources would flow to "weak" firms, the majority of fiscal resources flow to "strong" firms that do not need the support. The pandemic loans (cash grant) policy disbursed 6.43% (2.63%) of GVA in funding to firms. The total cost of saving "viable" firms is 0.53 (0.19) and "weak" firms is 0.45 (0.19) percent of GDP. Note that though the actual cost of bailing out "weak" firms is small, saving them is likely a poor use of funds because they are likely to struggle and fail after fiscal support ends. The remainder of funds are directed towards "strong" firms. The cash grant policy disburses over 2% of GDP to "strong" firms. Though the pandemic loan is costly in terms of disbursements, providing 5.45% of GDP to "strong" firms, one potential advantage is that these funds may be recovered in the future. If the 5.45% of GDP distributed to "strong" firms were to be fully recovered by repayments, the overall cost of the policy would fall to 0.98% of GDP and the fiscal bankruptcy multiplier would rise to 2.12/0.98= 2.16—a fiscally cost-effective policy.

Table 12 and Table 13 show that the pandemic loan and cash grant policies were untargeted across firm types. Focusing on the pandemic loan policy, Fig. 7 investigates whether there is any selection *within* firm type. Specifically, we compare the labor productivity (panels (a) and

[55] We do not show a column for failure rates of strong firms because these are zero by definition.

[56] Weak firms comprise 8.26% of all firms, which is less than the 9.53% of firms we estimate would fail in a non-COVID 2020 scenario (Table 3). As discussed above, the remaining 1.27% of firms that fail in our non-COVID scenario *survive* COVID because they are in sectors receiving positive demand shocks.

[57] Firms preserved by each policy can be calculated by subtracting the failure rate in each policy from the total number of firms in each subgroup (8.26% for weak firms and 7.28% for viable firms). For example, due to the cash grant policy, 8.26-6.30=1.96% of all firms were weak and saved. Therefore 1.96/8.26 = 23% of all weak firms were saved by the cash grant policy.

Table 12: The Distribution of Policy Support by Firm Type

	Firms that Survive COVID (Strong Firms)	**Firms Bankrupt Regardless of COVID (Weak Firms)**		**Firms Bankrupt Only in COVID Scenario (Viable Firms)**		**Total**
	(1)	(2)	(3)	(4)	(5)	(6)
	Funds Disbursed* (% GDP)	Failure Rate (% Firms)	Funds Disbursed* (% GDP)	Failure Rate (% Firms)	Funds Disbursed* (% GDP)	Funds Disbursed* (% GDP)
Benchmark Policy	0.00	8.26	0.00	0.00	0.77	0.77
Cash Grant	2.24	6.30	0.19	4.51	0.19	2.63
Pandemic Loans	5.45	4.75	0.45	2.94	0.53	6.43

Notes: Because Orbis does not cover the universe of firms, we calculate aggregate costs by scaling the total costs in Orbis by the inverse of the coverage ratio of Orbis (based on 2018 value added at the 1-digit NACE level). The numbers presented here are GDP-weighted averages.

* Unlike the other policies, the funds disbursed under the pandemic loan policy do not equal the fiscal cost, which depends on the rate of repayment and the distribution of losses between the government and the banking sector.

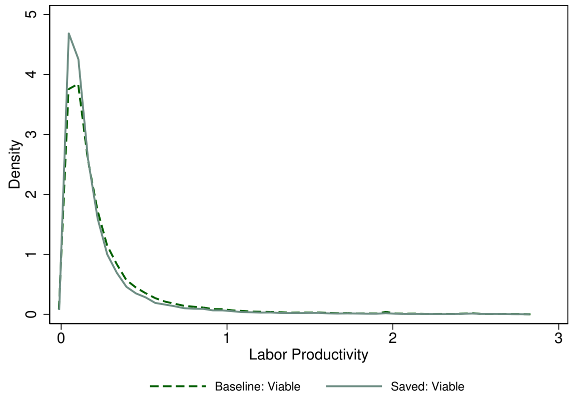
Table 13: Wages, Jobs and Loans Saved by Firm Type

	(1)	(2)	(3)	(4)
	Jobs Saved (% Emp)	Wages Saved (% GVA)	Net worth Preserved (% GVA)	Policy Cost* (% GVA)
	Firms Bankrupt Regardless of COVID (Weak Firms)			
Benchmark Policy	0.00	0.00	0.00	0.00
Cash Grant	1.10	0.48	1.44	0.19
Pandemic Loans	1.80	0.82	3.53	0.45
	Firms Bankrupt Only in COVID Scenario (Viable Firms)			
Benchmark policy	3.11	1.44	15.49	0.77
Cash Grant	1.53	0.66	3.52	0.19
Pandemic Loans	2.23	0.98	6.90	0.53

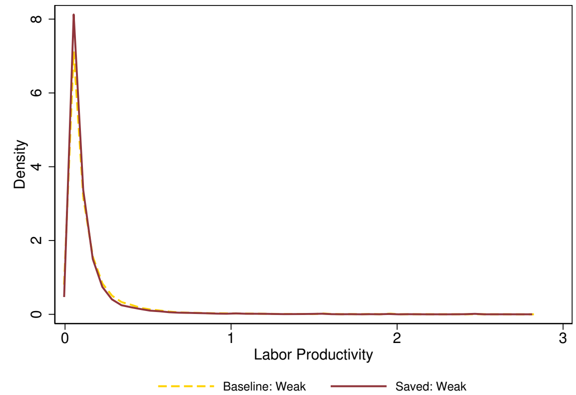
Notes: Because Orbis does not cover the universe of firms, we calculate aggregate costs by scaling the total costs in Orbis by the inverse of the coverage ratio of Orbis (based on 2018 value added at the 1-digit NACE level). The numbers presented here are GDP-weighted averages. * Unlike the other policies, the funds disbursed under the pandemic loan policy do not equal the fiscal cost, which depends on the rate of repayment and the distribution of losses between the government and the banking sector.

(b)) and initial cash-to-assets ratio (panels (c) and (d)) of all versus saved "viable" (panels (a) and (c)) and "weak" (panels (b) and (d)) firms. We see some evidence that saved "viable" firms have higher labor productivity, relative to the whole group; but see no such difference in the initial cash-to-assets ratio. Meanwhile, saved "weak" firms look virtually identical to the rest of their group in both labor productivity and the initial cash-to-assets-ratio.

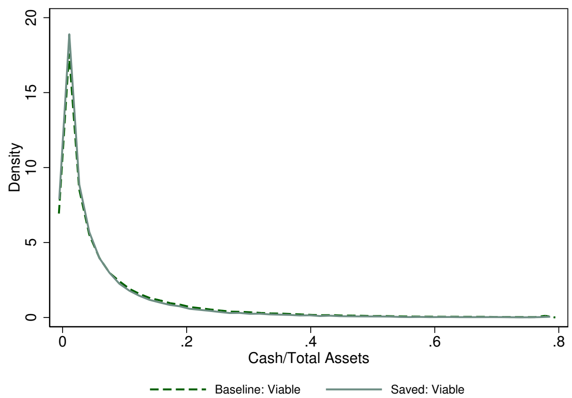
This section highlights how our framework can be used to provide insights on the cost and impact of various fiscal policies. While some were concerned that fiscal support would disproportionately benefit "weak" firms, our framework highlights a more nuanced message. Because policymakers lack full information and were pressed to respond quickly in the midst of the crisis, untargeted and costly policies were implemented. Through the lens of our framework, we predict that while these policies save many "viable" firms, they disburse the vast majority of funds to "strong" firms. A small number of "weak" firms are preserved as well, but they claim a small proportion of the funds dispersed.



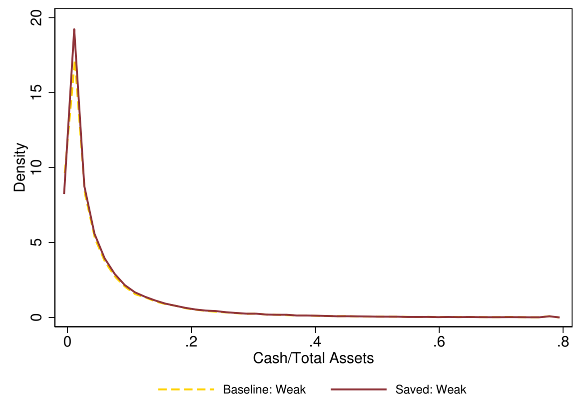
(a) Log Labor Productivity: Viable & Saved



(b) Log Labor Productivity: Weak & Saved



(c) Cash/Total Assets (%): Viable & Saved



(d) Cash/Total Assets (%): Weak & Saved

Figure 7: Baseline vs. Pandemic Loan Scenarios: Distributions

Notes: Depicted are the distributions of (a) log labor productivity for all viable firms under the baseline scenario and viable firms saved under the pandemic loan scenario, (b) log labor productivity for all weak firms under the baseline scenario and weak firms saved under the pandemic loan scenario, (c) initial cash-to-assets ratio for all viable firms under the baseline scenario and viable firms saved under the pandemic loan scenario, (d) initial cash-to-assets ratio for all weak firms under the baseline scenario and weak firms saved under the pandemic loan scenario.

Our findings therefore suggest that policy design is critical. Policymakers have options that may help reduce their overall fiscal burden. Take the pandemic loan policy as an example. The fiscal burden of this policy is lessened because "strong" firms are likely to repay, but a risk remains because some "viable" firms may not be able to repay loans. Instead, policymakers could couple immediate support with a mechanism by which fiscal authorities recoup some of the relief in future years from the best performing survivors—for example., via an excess profit tax (see Blanchard et al. (2020), Drechsel and Kalemli-Ozcan (2020), and Hanson et al. (2020) for similar recommendations).

6 Ex-post analysis of COVID

In this section, we compare excess failure rates under several COVID scenarios to those reported in Eurostat.[58] The ex-post (for most countries, estimated) excess failure rate is calculated using Eurostat data as the change in employer business failure rates between 2019 and 2020. In Table 14, we compare this ex-post excess failure rate with estimated excess failure rates (COVID- non-COVID) under different scenarios derived using (a) shocks calibrated with data that became available in later phases of the crisis (col. 3), and (b) realistic fiscal policy responses (col. 4 and 5) in order to assess whether shocks, policy, or both can bring our baseline estimates more closely in line with the ex-post data.[59]

Column (2) of Table 14 shows that the baseline excess failure rate is, on average, 5.48 percentage points higher (6.11 in col. 2 less 0.63 in col. 1) than the ex-post excess failure rate (6.17 percentage points when Finland is excluded). This large gap is primarily due to two baseline assumptions—we model a single 8 week lockdown period and assume no government support is given to firms. Columns (3) through (5) modify these assumptions, bringing estimated excess failures rates closer in line with ex-post excess failure rates.

In column (3), the aggregate demand shock is calibrated using the realized OECD GDP growth between 2019 and 2020 (instead of the IMF WEO *forecast* of each country's 2020 GDP growth), and the single 8 week lockdown period is replaced with sectoral shocks that are allowed to vary over the course of 2020 with country-specific lockdown intensity.[60] Changing how shocks are calibrated lowers the difference between the estimated and ex-post excess failure rates to 4.78 percentage points (5.53 percentage points when Finland is excluded).

In column (4), we calibrate shocks as in column (3) and introduce the blanket pandemic loan policy detailed in Section 5.4.1. This scenario lowers the estimated excess failure rates to -2.52 percentage points. The difference between the estimated and ex-post excess failure rates falls to -3.15 percentage points. In column (5), we calibrate shocks as in column (3), and also calibrate government support to the type, amount, and take-up rate (when available) of each

[58]It is worth noting that the 2020 Eurostat failure rates used are are still provisional for many countries because data on businesses is often finalized with roughly a three year lag. See Table 29 in Appendix I.

[59]Please note that in this section we exclude Hungary from the analysis because they report a break in their failure rate series in 2020. Additionally, although we include Finland in the analysis, we report overall averages with and without Finland because they report a 2020 failure rate that is 20.29 percentage points above the 2019 figure, which we view as surprisingly high.

[60]Specifically, we use two series that were produced during the pandemic—the Oxford Government Response Tracker's (OxCGRT) stringency index and Google mobility data—to generate country-specific, weekly measures of lockdown intensity. The lockdown stringency index can be obtained from Oxford Government Response Tracker and the mobility data from Google's COVID-19 Mobility Reports. Because the OxCGRT index tracks government containment measures, we map it to our sectoral labor supply ($\hat{x}_s$) and productivity ($\hat{A}_s$) shocks. The Google mobility data tracks shopping activity, which we map to our sector-specific demand shock ($\tilde{\xi}_s$). We normalize both indexes to vary from 0 to 1 and interact them with the appropriate shocks to obtain new shocks that vary by country, sector and week.

country.[61] Our model estimates the overall excess failure rate to be -0.78, percentage points, which brings the difference between the estimated and ex-post excess failure rates down to -1.41 percentage points (-0.79 percentage points when Finland is excluded).

Taken together, Table 14 suggests that characterizing policy accurately is important for bringing our estimated excess failure rates closer in line with recent estimates of excess failure rates in 2020. This finding is reassuring given the policy implications of the paper: in the absence of policy support, the excess failure rates could have been very high in 2020.

Table 14: Exploiting Ex-Post Information

	(1) Δ Eurostat Failure Rates (2019-2020)	(2) Baseline	(3) Lockdown Intensity & OECD Δ GDP	(4) Column (3) & Pandemic Loans	(5) Column (3) & Calibrated Policy
Czech Republic	-0.15	2.61	1.34	-3.12	-2.34
Finland	20.29	4.26	3.45	-4.09	0.89
France	1.52	5.35	4.77	-3.72	-2.19
Italy	-0.80	10.55	8.83	0.47	0.42
Poland	-2.55	4.20	1.58	-4.04	-3.63
Portugal	-0.61	4.10	5.01	-3.39	1.95
Romania	-3.65	2.56	1.62	-5.23	1.62
Slovak Republic	-8.99	3.11	1.38	-3.13	0.18
Slovenia	1.11	2.95	2.16	-2.66	0.39
Spain	0.24	3.87	5.21	-3.03	0.34
Average (all)	0.63	6.11	5.41	-2.52	-0.78
Average (excluding FI)	-0.05	6.17	5.48	-2.47	-0.84

Notes: Reports the Eurostat change in failure rates between 2019 and 2020 (col. 1); and excess failure rates in COVID under the following scenarios: baseline (col. 2), variable lockdown intensity and observed country-level OECD GDP growth rates (2019-2020) (col. 3), column (3) coupled with blanket pandemic loans (col. 4), and column (3) coupled with policies calibrated to actual policies implemented by each country, with take-up rates accounted for when possible. Failure rates are first calculated at the 1-digit NACE level and aggregated to the country level using 2018 sector GVA as weights. Failure rates are aggregated across countries using GVA as weights. Note that Hungary is excluded because it experienced a break in the time series in 2020.

7 Conclusion

In this paper, we introduce a framework to study the impact of firm financial frictions on SME failures in the presence of shocks to firms' liquidity. Firms fail when, as a result of shocks, they are unable to cover input costs and financial expenses, and they cannot borrow to cover these expenses. We combine the model with detailed firm level balance sheet data that enables us to characterize a baseline distribution of firm outcomes prior to any scenario.

Using firm level data for SMEs in a sample of 11 European countries, we first use our framework to implement a "typical" year scenario, in which firms face modest shocks. We find that, in 2017-2019, the mean forecast error at the country-sector level is only 0.69 percentage points. We also show that firms predicted to fail are less productive and profitable, grow

[61] Details on how this is done is in Appendix K.

slower, have less cash on hand, and are more leveraged than those predicted to survive, which is consistent with predictions from the empirical and theoretical literature.

We then apply our framework to COVID-19 to illustrate the impact of a large cash flow shock on SME failures. First, we consider a baseline scenario, absent government support, and estimate a 6.01 percentage point excess SME failure rate. We highlight the importance of the interaction between exposure to sectoral shocks and firm financial constraints in explaining the observed heterogeneity in cross-sector and cross-country excess SME failure rates. We also show that "viable" firms are similar to "strong" firms in terms of labor productivity, profitability, and growth, but similar to "weak" firms in that they are cash poor and highly leveraged. In short, firms with good fundamentals can fail in crises and recessions. In evaluating fiscal support measures, we find that while cash grants and pandemic loans save many SMEs from failure, they do so in an untargeted fashion and at a high cost. Both policies primarily save "viable" firms, but also save some "weak" firms, though at a low fiscal cost. The high policy cost is due to the vast majority of funds disbursed being channeled to "strong" firms that do not need the support.

Our work has important policy implications. Government programs can help saving productive firms during large liquidity shocks, however, they need to be targeted to reduce their fiscal burden. This finding underlines the importance of developing institutional infrastructures, where granular firm level data can be used real-time by governments to target firm support programs.

Appendix

A Elasticities, Shocks and Orbis Coverage

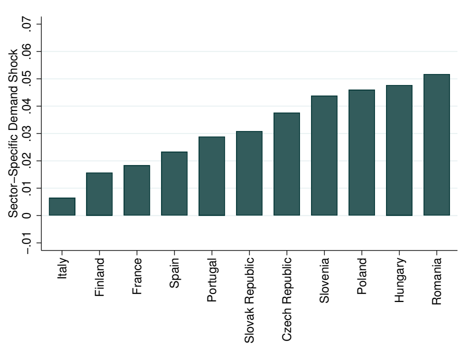
Labor and material elasticities (β_s and γ_s) are calculated at the 2-digit NACE level for each country. Table 15 reports the cross-country mean and standard deviation of these elasticities at the one-digit NACE level.

Fig. 8 depicts the average of (a) total demand and (b) sectoral productivity shocks at the country level for our typical year scenario (2017-2019).

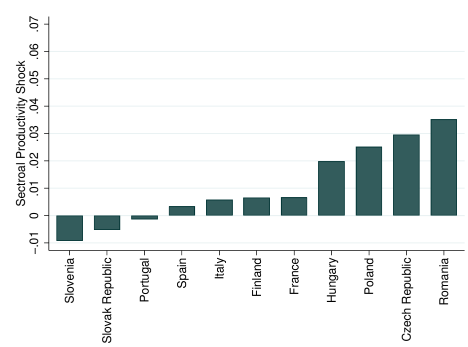
Table 15: Output Elasticities

	Labor (β_s)		Materials (γ_s)	
	Mean	SD	Mean	SD
Agriculture	0.18	0.07	0.46	0.13
Mining	0.23	0.21	0.27	0.16
Manufacturing	0.17	0.07	0.54	0.13
Electric, Gas & Air Con	0.06	0.02	0.58	0.09
Water & Waste	0.22	0.08	0.26	0.14
Construction	0.19	0.07	0.33	0.09
Wholesale & Retail	0.08	0.03	0.76	0.05
Transport & Storage	0.21	0.10	0.22	0.14
Accom. & Food Service	0.26	0.05	0.33	0.14
Info. & Comms	0.23	0.10	0.17	0.10
Real Estate	0.14	0.04	0.33	0.09
Prof., Sci., & Technical	0.27	0.11	0.26	0.15
Administration	0.35	0.24	0.19	0.15
Education	0.42	0.11	0.14	0.11
Health & Social Work	0.46	0.12	0.16	0.09
Arts, Ent., & Recreation	0.23	0.11	0.16	0.12
Other Services	0.30	0.19	0.35	0.17

Notes: Elasticities are calculated at the 2-digit NACE level as the weighted average of the labor cost share of revenue (β_s) and material cost share of revenue (γ_s), where the weights are given by firm revenue. These elasticities are calculated for countries where labor and material costs are reported separately (Czech Republic, Finland, France, Hungary, Italy, Poland, Portugal, Romania, Slovakia, Slovenia, and Spain). The table reports the cross-country mean and standard deviation of the elasticities at the 1-digit NACE level.



(a) Total Demand Shock



(b) Sectoral Productivity Shock

Figure 8: Shocks by Country: Typical Year Scenario (2017-2019)

Notes: Depicts the typical year scenario (2017-2019) (a) total demand and (b) sectoral productivity shocks by country. The height of each bar represents the simple average of the shock across sector-years in each country.

Table 16 reports the aggregate revenue coverage for the countries in our sample, both for all firms and SMEs specifically in 2018. SMEs are defined as firms with less than 250 employees in both data sources, OECD and Orbis. Using raw Orbis data, our coverage ranges from 34.0% in France to 55.7% in Italy.[62] Focusing on SMEs, our coverage ranges from 33.1% in France

[62]To obtain coverage rates we sum up all firm (and, separately, SME) revenue in Orbis by 1-digit NACE sector and merge it with 1-digit NACE sector total (and SME) revenue reported in the OECD's SDBS Business

to 66.7% in Slovak Republic. Even after imposing additional data requirements for analysis, such as availability of intermediate costs, our data cover at least 30% of the aggregate revenue of SMEs in our sample of countries.

Table 16: Orbis Coverage (2018)

	% of OECD Revenue	
	All Firms	SMEs
Czech Republic	49.1	37.0
Finland	53.2	52.9
France	34.0	33.1
Hungary	43.7	39.9
Italy	55.7	64.7
Poland	39.6	36.2
Portugal	52.1	62.6
Romania	51.5	37.5
Slovak Republic	50.5	66.7
Slovenia	46.1	53.6
Spain	47.5	62.1

Notes: OECD revenue (all firms and SMEs) in 2018 is obtained from the Structural Business Statistics Database. The SBSD provides data for a subset of sectors—for most countries the covered NACE 1-digit sectors are B, C, D, E, F, G, H, I, J, L, M, and N. Only sectors covered in both the OECD and Orbis data are used in calculating coverage statistics. To calculate coverage, Orbis revenue (all firms and SMEs) is summed and divided by the total revenue (all firms and SMEs) reported by OECD. The coverage rates are computed using cleaned Orbis data. Additional cleaning is done to generate the analysis data, including conditioning on variables needed to compute the failure condition. SMEs are defined as firms with less than 250 employees in both OECD and Orbis data.

Demography Indicators. Keeping sectors covered in the Orbis and OECD data (for most countries the covered sectors are B, D, D, E, F, G, H, I, J, L, M,and N), we then aggregate the Orbis and OECD data to the country level and calculate the coverage rates for all firms and SMEs.

Online Appendix

B Sector-Level Forecast Errors in Typical Years

Table 17 reports the official Eurostat failure rates (col. 1), our forecasted typical year failure rates (col. 2) and the forecast error (col.3) at the one-digit sector level. The table shows that our framework does reasonably well in forecasting actual failure rates at the sector level in normal times, with forecast errors ranging from -2.77 in Construction to 4.31 in Mining.

Table 17: Failure Rates Comparison at the Sector Level (2017-2019)

	(1) Eurostat Failure Rate	(2) Forecasted Failure Rate	(3) Δ (Forecasted - Eurostat)
Mining	6.51	10.82	4.31
Manufacturing	7.05	8.01	0.97
Electric, Gas & Air Con	7.76	9.31	1.55
Water & Waste	6.16	7.51	1.35
Construction	10.92	8.16	-2.77
Wholesale & Retail	9.25	8.84	-0.41
Transport & Storage	9.17	8.08	-1.09
Accom. & Food Service	10.79	13.21	2.42
Info. & Comms	9.40	10.44	1.04
Real Estate	10.30	11.75	1.45
Prof., Sci., & Technical	8.71	10.32	1.61
Administration	10.11	9.71	-0.39
Education	9.09	11.31	2.22
Health & Social Work	6.82	8.10	1.28
Arts, Ent., & Recreation	10.30	11.74	1.44
Other Services	9.54	13.11	3.57

Notes: Eurostat failure rates are obtained from the Structural Business Statistics for employer businesses at the (country x 1-digit NACE x year) level. Failure rates are forecasted by combining Orbis firm level balance sheet data with sector-specific demand and labor productivity shocks calculated using Eurostat national accounts at the 1-digit NACE level, and aggregate demand shocks measured as quarterly GDP growth from OECD.Stat. The liquidity criterion is evaluated for each firm at the end of the year. The table shows (1) official Eurostat and (2) forecasted failure rates, as well as the (3) the forecast error (i.e. Forecasted-Eurostat failure rate) at the country level. The (country x sector x year) observations are first aggregated to the (sector x year) level using (country × sector) GVA as weights. The observations are then aggregated to the sector level by taking a simple average over time (2017-2019).

C Firm Entry During COVID

Currently, our focus is on excess failure rates in COVID, which we define as failure rate in COVID - failure rate in non-COVID. If we were to think instead about the excess net entry rate in COVID, we would define it as follows:

$$\begin{aligned} \text{Excess Net Entry}^{\text{COVID}} =& (\text{Entry Rate}^{\text{COVID}} - \text{Exit Rate}^{\text{COVID}}) - \\ & (\text{Entry Rate}^{\text{non-COVID}} - \text{Exit Rate}^{\text{non-COVID}}) \end{aligned} \tag{33}$$

Currently, we are implicitly assuming that:

$$\text{Entry Rate}^{\text{COVID}} = \text{Entry Rate}^{\text{non-COVID}} \tag{34}$$

Under such an assumption:

$$\begin{aligned}\text{Excess Net Entry}^{\text{COVID}} &= -(\text{Exit Rate}^{\text{COVID}} - \text{Exit Rate}^{\text{non-COVID}}) \\ &= -\text{Excess Failure Rate}^{\text{COVID}}\end{aligned} \tag{35}$$

An open question is the direction of bias that our assumption potentially introduces. Official Eurostat entry rates are available between 2017 and 2020, subject to the caveats discussed in 29 regarding the 2020 data. In Table 18, we compare the entry rate in normal times (defined as the average firm entry rate between 2017 and 2019), which we consider to be a proxy for non-COVID entry rates, with entry rates in 2020. The table shows that, with the exception of Finland, entry rates in 2020 (COVID) were lower than in normal times (non-COVID), which would make excess net entry smaller and our excess failure rate larger.

Table 18: Eurostat Employer Firm Entry Rates (%)

	(1)	(2)	(3)
	2017-2019	2020	Δ
Czech Republic	7.86	6.30	1.56
Finland	11.01	12.94	-1.93
France	11.27	10.47	0.81
Hungary	16.37	9.21	7.16
Italy	8.41	7.27	1.14
Poland	13.68	9.64	4.04
Portugal	11.20	8.88	2.32
Romania	11.54	10.02	1.52
Slovak Republic	12.07	6.30	5.77
Slovenia	9.24	7.36	1.88
Spain	9.67	7.67	1.99

Notes: Reports the average employer firm entry rate in 2017-2019 (col. 1), 2020 (col. 2), and the difference between the two (Δ, col. 3).

D Granularity of Shocks

D.1 Idiosyncratic shocks and forecast performance

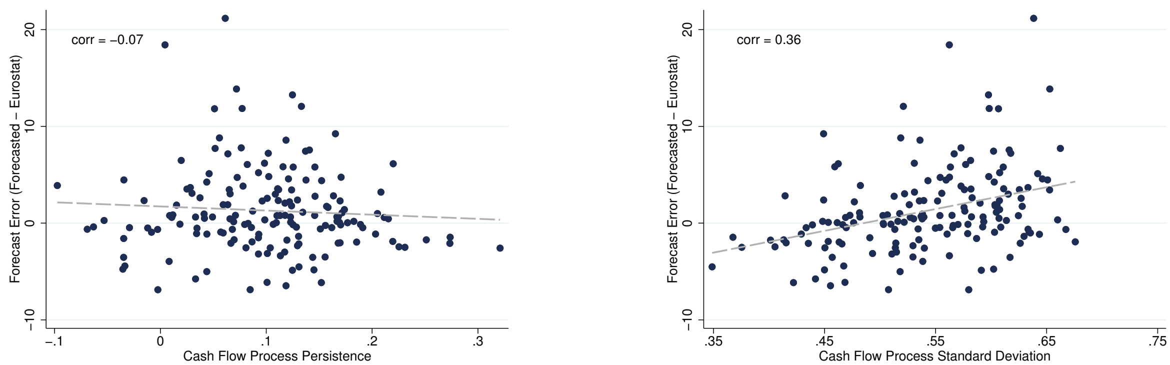
To study the relationship between forecast errors and idiosyncratic shocks, Figure 9 plots the relationship between the county-sector level forecast errors and the (a) persistence and (b) standard deviation of the firm level cash flow process.

Specifically, we use firm level data between 2010 and 2018 to estimate:

$$CF_{i,t} = \alpha_i + \gamma_{s,t} + \beta CF_{i,t-1} + \varepsilon_{i,t}$$

for each country × one-digit sector, where $CF_{i,t}$ is the cash flow of firm i in year t, α_i is a firm fixed effect, and $\gamma_{s,t}$ is a 4-digit × year fixed effect. β is a proxy for persistence and the standard deviation of ε_{it} is a proxy for the volatility of the cash flow process.

Figure 9 shows that there is a small negative correlation between the forecast error and the persistence of the cash flow process, and a moderate positive correlation between the forecast error and the volatility of the cash flow process. The figure shows that idiosyncratic shocks, especially their volatility, may help explain part (but not all) of the forecast error.



(a) Cash Flow Persistence

(b) Cash Flow Standard Deviation

Figure 9: Forestcast Errors & Cash Flow Process

Notes: Depicts the relationship between the average forecast error (estimated - Eurostat) at the country-sector level (2017-2019) and the (a) persistence and (b) standard deviation of cash flow process. Using firm level data between 2010 and 2018 to estimate $CF_{i,t} = \alpha_i + \gamma_{s,t} + \beta CF_{i,t-1} + \varepsilon_{i,t}$ for each country × one-digit sector, where $CF_{i,t}$ is the cash flow of firm i in year t, α_i is a firm FE and $\gamma_{s,t}$ is a 4-digit × year FE. β is a proxy for persistence and the standard deviation of ε_{it} is a proxy for the volatility of the cash flow process.

D.2 Sectoral shock granularity

Table 19 shows the importance of granular sector level shocks for our COVID findings. Column (1) shows our baseline estimates, where shocks are measured at the 4-digit NACE sector level. Columns (2) through (4) show the effect of aggregating these shocks to the 3-digit, 2-digit and 1-digit level, respectively. As can be seen, excess failure rates become considerably smaller because aggregation obscures the granular shocks faced by many firms during COVID.

Table 19: Aggregating our sectoral COVID shocks

Excess Failure Rates (COVID – non-COVID)

	Baseline (1)	3 digit (2)	2 digit (3)	1 digit (4)
Czech Republic	2.56	2.37	1.85	1.33
Finland	4.18	3.72	2.43	1.29
France	5.27	4.92	4.10	3.13
Hungary	2.77	2.39	1.78	1.25
Italy	10.30	9.89	8.69	7.00
Poland	5.50	5.48	4.06	3.87
Portugal	4.02	3.51	2.12	1.59
Romania	2.37	2.21	1.63	0.95
Slovak Republic	3.02	2.66	2.36	1.62
Slovenia	2.98	2.76	1.76	1.02
Spain	3.75	3.45	2.27	1.77
All	6.01	5.68	4.65	3.67

Notes: Reports the excess failure rate (COVID - non-COVID) under four scenarios with different aggregation of COVID shocks — (1) the baseline (4-digit NACE level sectoral shocks), (2) shocks at the 3-digit NACE level, (3) shocks at the 2-digit NACE level and (4) with shocks at the (1) digit NACE level. When aggregating sectoral shocks we weight by each sector's value added. Sector excess failure rates (Δ) are first calculated at the 1-digit NACE level for each country, and then aggregated across countries using (country x sector) GVA as weights. The last row is the sector GVA weighted average.

E Flexible Prices

The next two tables highlight the effects of introducing price flexibility into our framework. Details regarding the implementation are in Appendix E.1. In the flexible price exercise, we model shocks at the sectoral level, as we do in the baseline. Consequently, price changes occur at the 4-digit NACE level. Moreover, although we allow firm prices to change, we continue to assume that materials prices and wages are fixed. Because prices across 4-digit sectors vary, the effect on firm outcomes depends on the cross-sector elasticity of substitution (η). Here we consider two cases—a reasonably realistic case where $\eta = 1$, a case with extreme complementarity where $\eta = 0.2$ and cases covering substitutes with $\eta = 2$.

Column (1) of table 20 reports our baseline excess failure rates, columns (3) to (5) reports the flexible price scenario with $\eta = 0.2$, $\eta = 1$ and $\eta = 2$ respectively. In all flexible price scenarios, we find higher excess failure rates. Table 21 explains these findings.

In essence, flexible prices are helpful or harmful to firms depending on whether they face increasing costs or low demand. To showcase this, we divide firms into three groups: those that are demand constrained (i.e. their desired labor is less than that allowed by lockdown orders) and two sub-groups of supply constrained firms. While all supply constrained firms

Table 20: Allowing Flexible Prices

Excess Failure Rates (COVID – non-COVID)

	Baseline (1)	($\eta = 0.2$) (2)	($\eta = 1$) (3)	($\eta = 0.2$) (4)	($\eta = 2$) (5)
Czech Republic	2.56	4.94	5.13	4.96	5.67
Finland	4.18	8.54	10.74	8.65	11.00
France	5.27	14.97	17.08	15.11	17.67
Hungary	2.77	5.81	7.11	5.85	7.97
Italy	10.30	18.69	20.72	18.96	21.54
Poland	5.50	9.65	11.10	9.54	11.40
Portugal	4.02	11.03	12.76	10.90	12.86
Romania	2.37	4.29	3.98	4.12	5.01
Slovak Republic	3.02	6.39	7.21	6.29	7.53
Slovenia	2.98	9.30	11.93	9.24	11.95
Spain	3.75	9.31	10.59	9.36	10.63
All	6.01	13.37	15.11	13.48	15.63

Notes: Reports the excess failure rate (COVID - non-COVID) under scenarios with flexible prices — (1) shows our baseline with fixed prices, (2) with a cross-sector elasticity of $\eta = 0.2$; (3) a scenario with flex prices and a cross-sector elasticity of $\eta = 1$; and (4) a scenario with flexible prices and a cross-sector elasticity of 2. When aggregating sectoral shocks we weight by each sector's value added. Sector excess failure rates (Δ) are first calculated at the 1-digit NACE level for each country, and then aggregated across countries using (country x sector) GVA as weights. The last row is the sector GVA weighted average.

are constrained in their labor choices by lockdowns $\hat{n}_s > \hat{x}_s$, some are more affected by low demand and opt to lower their prices in response. These firms constitute our second group. The third group of firms are highly supply constrained and as such, opt to raise prices. Outcomes for these three groups are shown in columns (1), (2) and (3) respectively of Table 21.

The top portion of the table focuses on week 9 in our baseline 2020 COVID scenario. This week marks the beginning of our lockdown, and shows how each group fares during this phase of 2020. The first block shows how prices change for each group of firms based on four scenarios (fixed prices, flexible prices with $\eta = 0.2$, $\eta = 1$ and $\eta = 2$. Consistent with our hat algebra, these changes are gross changes: $\hat{P} = P'/P$ so that values above 1 indicate a rise in prices, and values below 1 indicate a reduction in prices. By construction, in our benchmark, fixed price scenario prices do not change. In the flexible price scenarios demand constrained firms lower their prices, as do the mildly supply constrained firms. The latter price reduction is by construction (i.e. how we define this group of firms). Similarly, by construction, the extreme supply constrained firms raise their prices.

The next block of Table 21 reports firms output. As expected, relative to our fixed price benchmark, output is higher for the firms that lower their prices (cols. 1 and 2) and lower for the firms that raise their prices (col. 3). Critically for understanding why failure rates rise, revenue for firms that lower prices in the case where $\eta = 1$ is unchanged, and falls for the case of $\eta = 0.2$. While revenue rises when elasticities of demand are above 1, the key insight is that *all* firms lowering prices has a smaller effect on a particular firms' revenues

Table 21: Understanding the Flexible Price Results

			(1)	(2)	(3)
	Fixed or flexible?	Price Elasticity of demand (η)	Demand Constrained	Supply Constrained *Lowered Prices*	*Raised Prices*
		Changes in Week 9			
Prices *(above 1 = rise)*	*Fixed*	.	1.00	1.00	1.00
	Flexible	0.20	0.63	0.69	1.04
	Flexible	1.00	0.77	0.88	1.07
	Flexible	2.00	0.83	0.95	1.08
Output *(above 1 = rise)*	*Fixed*	.	0.35	0.55	1.18
	Flexible	0.20	0.36	0.56	1.06
	Flexible	1.00	0.55	0.64	1.09
	Flexible	2.00	0.64	0.65	1.10
Revenue *(above 1 = rise)*	*Fixed*	.	0.35	0.55	1.18
	Flexible	0.20	0.19	0.41	1.09
	Flexible	1.00	0.35	0.55	1.18
	Flexible	2.00	0.47	0.62	1.19
COGS *(above 1 = rise)*	*Fixed*	.	0.20	0.31	1.25
	Flexible	0.20	0.19	0.41	1.09
	Flexible	1.00	0.35	0.55	1.18
	Flexible	2.00	0.47	0.62	1.19
Cash Flow *(fraction of revenue)*	*Fixed*	.	0.33	0.45	0.24
	Flexible	0.20	0.12	0.14	0.34
	Flexible	1.00	0.22	0.32	0.35
	Flexible	2.00	0.30	0.38	0.34
		End of the year			
Change in Failure Rates *(COVID – non-COVID)*	*Fixed*	.	14.88	13.07	1.42
	Flexible	0.20	38.11	38.97	3.34
	Flexible	1.00	31.09	29.75	4.23
	Flexible	2.00	26.48	22.96	4.85

Notes: Reports a variety of outcomes for three groups of firms in week 9 of our 2020 COVID scenario. The firm groups are those that are "demand constrained", those that are "supply constrained" but nonetheless lower prices and those that are "supply constrained" but raise prices. These classifications are made in the $\eta = 1$ case and kept the same across different elasticity choices so that composition effects do not affect the comparisons. Changes in prices, output and revenue all only vary at the 4-digit sectoral level and are averaged for each group weighting each sector by gross value added. Cost of goods sold (COGS) and cash flow are averaged with equal weights within 1-digit NACE sectors and then using gross value added as weights to go from the 1-digit sector x country level to an overall average.

than would emerge if only that firm were lowering its price. Thus for firms *on average* relative prices are not changing and therefore revenue on average is not changing. [63] If revenue is not changing on average but prices are lower, then per-unit markups must be lower on average. This can be seen by comparing revenue with the cost of goods sold (COGS) – in our fixed price benchmark scenario demand constrained firms (col. 1) face revenue falls of 65% (1-0.35), but a much larger 80% decline in their COGS – leading to a rise in the average per-unit markup

[63] Recall in our partial equilibrium framework that overall expenditures on goods $\hat{PD}$ evolves exogenously.

for firms In every flexible price scenario however, markups are preserved at pre-COVID levels and as such revenue and COGS decline by similar proportions. This leads to larger declines in cashflow relative to the fixed price benchmark and higher failure rates.

Importantly, even firms that raise prices in lockdown end up with higher excess failure rates under flexible prices than fixed prices. This occurs because these firms only get the benefits of raising prices when their production is constrained, which we assume occurs only in weeks 9-16 of 2020. Afterwards, we assume that workplace restrictions end and demand slowly reverts back to pre-COVID levels. During the period of recovering demand, all firms become demand constrained and are at risk of being trapped in price wars with other firms. Therefore, even the firms most likely to benefit from price flexibility in lockdown are harmed by it during the rest of the year.

E.1 Details to Implement Flexible Prices

Firms in a sector s now solve the following profit maximization problem:

$$\begin{aligned} \max y'_{is}, n'_{is}, m'_{is} \quad & p'_{is} y'_{is} - w_s n'_{is} - p^m_s m'_{is} \\ s.t. y'_{is} &= z_{is} k^{\alpha_s}_{is} (A'_s n'_{is})^{\beta_s} (m'_{is})^{\gamma_s} \\ n'_s &< \hat{x}_s n_s \end{aligned}$$

We assume that sector level demand elasticities (ρ_s) are constant which will deliver prices that preserve pre-COVID markups. This leads to 100% pass-through of COVID shocks to prices: $p'_{is} = \mu_{is} mc'_{is}$, where μ_{is} is the pre-COVID markup and mc'_{is} is marginal cost in COVID. In making this assumption, we focus on the polar opposite of our fixed price assumption, which better enables us to benchmark the difference between fixed prices and flexible prices. In what follows, we define the gross change in a variable as $\hat{x} = x'/x$. Because shocks only vary at the sectoral level, our hat variables will also only vary at the sectoral level. As such, we omit i subscripts unless applied to actual firm level data.

Now that prices can change our change in relative demand equation (Eq. (7) in the text) becomes:

$$\widehat{pd}_s = \hat{\xi}^{\eta} \left(\frac{\hat{p}_s}{\hat{P}} \right)^{1-\eta} \widehat{PD} \tag{36}$$

where under the assumption of a symmetric initial equilibrium:

$$\hat{P}^{1-\eta} = \frac{1}{S}\sum_{\sigma} \hat{\xi}_{\sigma}^{\eta} \hat{p}_{\sigma}^{1-\eta}$$

Labor constraint does not bind In this case we have that $\hat{n}_s = \hat{m}_s = \hat{p}_s \hat{y}_s$. Plugging this into the production function gives us:

$$\begin{aligned}
\hat{n}_s &= \hat{m} = \widehat{pd}_s \\
\hat{y}_s &= \hat{A}_s^{\beta_s} \left(\widehat{pd}_s\right)^{\beta_s+\gamma_s} \\
\hat{p}_s &= \hat{A}_s^{-\beta_s} \left(\widehat{pd}_s\right)^{1-\beta_s-\gamma_s}
\end{aligned} \tag{37}$$

Variable profits are given by:

$$\pi'_{is} = pd_{is} \widehat{pd}_s \left(1 - s_{ni} - s_{mi}\right)$$

where recall that s_{ni} and s_{mi} are the labor and materials shares of sales.

Labor constraint binds In this case we have $\hat{n}_s = \hat{x}_s$ and then:

$$\begin{aligned}
\hat{m}_s &= \widehat{pd}_s \\
\hat{y}_s &= \left(\hat{A}_s \hat{x}_s\right)^{\beta_s} \left(\widehat{pd}_s\right)^{\gamma_s} \\
\hat{p}_s &= \left(\hat{A}_s \hat{x}_s\right)^{-\beta_s} \left(\widehat{pd}_s\right)^{1-\gamma_s}
\end{aligned} \tag{38}$$

Variable profits are given by:

$$\pi'_{is} = pd_{is} \left(\widehat{pd}_s (1 - s_{mi}) - s_{ni} \hat{x}_s\right)$$

which can then be plugged into Equations 26 and 27 in the paper to evaluate firm failures.

Implementation When $\eta = 1$ we can solve this system using the same method as our benchmark cases because $\widehat{pd}_s = \widehat{PD} \tilde{\xi}_s^{\eta}$. When $\eta \neq 1$, $\widehat{pd}_s$ is a function of all sectoral prices. For cases where $\eta \neq 1$ we adopt the following two numerical procedures covering cases where $\eta < 1$ and $\eta > 1$ respectively. These both deliver exact non-linear solutions (up to a convergence criteria of 10^{-6}).

For $\eta < 1$:

1. We start by guessing the vector of sectoral prices $\underline{\hat{p}}^{(0)}$.

2. We then produce a price index $\hat{P}^{(0)}$ and solve for sector level demands $\hat{pd}_s^{(0)}$ using Equation 36.

3. Then taking sector level demand as given we update sector level prices using Equation 37 if labor is not constrained or Equation 38 if labor is constrained. This gives us a new vector of prices $\underline{\hat{p}}^{(1)}$

4. Repeat steps 2 and 3 until the change in prices is very small. We use $(\underline{\hat{p}}^{(n)} - \underline{\hat{p}}^{(n-1)})'(\underline{\hat{p}}^{(n)} - \underline{\hat{p}}^{(n-1)}) < 10^{-6}$

For $\eta > 1$:

1. We start by guessing the vector of sectoral expenditures $\underline{\hat{pd}}^{(0)}$.

2. Then taking sector level demand as given we update sector level prices using Equation 37 if labor is not constrained or Equation 38 if labor is constrained. This gives us a vector of prices $\underline{\hat{p}}^{(0)}$

3. We then produce a price index $\hat{P}^{(0)}$ and solve for a new vector of sector level expenditures $\underline{\hat{pd}}^{(1,*)}$ using Equation 36.

4. To prevent unstable oscillation in our guesses, we update sector level expenditures using $\underline{\hat{pd}}^{(1)} = 0.8\underline{\hat{pd}}^{(0)} + 0.2\underline{\hat{pd}}^{(1,*)}$

5. Repeat steps 2 and 3 until the change in sectoral final expenditures is very small. We use $(\underline{\hat{pd}}^{(n)} - \underline{\hat{pd}}^{(n-1)})'(\underline{\hat{pd}}^{(n)} - \underline{\hat{pd}}^{(n-1)}) < 10^{-6}$

F Redistributing Demand as firms fail

By having a single assessment period at the end of the year we avoid the need to reallocate expenditure between firms that have exited and surviving firms. However, for multiple assessment periods, an adjustment to the level of demand facing each surviving firm needs to be made as their competitors exit. We implement the following method, which avoids requiring estimates of sector-level price elasticities.

Assumption 1 *No selection on exit based on prices: We assume that the price distribution of firms that exit is the same as the overall price distribution in each sector. As such (from Equation 5):*

$$P_s' = \left(\frac{1}{\mathcal{N}_s'}\int_0^{\mathcal{N}_s'} p_{is}^{1-\rho_s}\right)^{\frac{1}{1-\rho_s}} = \left(\frac{1}{\mathcal{N}_s}\int_0^{\mathcal{N}_s} p_{is}^{1-\rho_s}\right)^{\frac{1}{1-\rho_s}} = P_s$$

From Equation 5 and the assumption of symmetry in the benchmark period, we also can derive the following expression for the *overall* price index $\hat{P}$:

$$\hat{P} = \frac{1}{S}\left(\sum_{s} \hat{\xi}_s^{\eta} \hat{\mathcal{N}}_s\right)^{1/(1-\eta)} \tag{39}$$

This leads to an adjusted Equation 7:

$$\hat{d}_{is} = \frac{\hat{\xi}_s^{\eta} \hat{\mathcal{N}}_s}{\sum_{\sigma} \hat{\xi}_s^{\eta} \hat{\mathcal{N}}_s / S} \widehat{PD}$$

This equation can be easily implemented in a sequential manner. For each period immediately following the assessment period, the set of firms in each sector is updated and demand is redistributed among all firms based on the relative exit rates in this equation. We implement this methodology in Table 5 when we consider alternative financing methods to credit lines.

However, when we varied the assessment period in Table 4 we deliberately did *not* make this demand reallocation adjustment. The reason was we wanted to showcase the *direct* effects on firms of limiting the length of time they have to recover temporary cash shortfalls. Table 22 shows the effect of also implementing the demand reallocation.

Column (1) reports our baseline scenario, where failure is assessed at the end of 2020. Columns (2) and (3) show the effect of making two assessments in 2020, columns (4) and (5) of making a quarterly assessments, and so on up to columns (8) and (9) for weekly assessments. The left-most columns in each section (col. 2, 4, 6 and 8) show the unadjusted numbers (also shown in Table 4) and the right-most columns in each section (col. 3, 5, 7 and 9) show the effect of accounting for demand-redistribution as firms fails. Focusing on the overall results (the last row) it is clear that adding this demand reallocation mechanism lowers failure rates on average by 1-2 percentage points.

G Introducing I-O linkages

We extend our framework to incorporate input-output (I-O) linkages and operationalize it using data from the 2014 World Input Output Database (WIDO). This I-O data only has 56 sectors, so we average our 4-digit NACE shocks to match these sectors. Within each sector, our lockdown shocks $\hat{x}_s$ and productivity shocks $\hat{A}_s$ remain unchanged. Our sectoral demand shock is then adjusted to impact only final demand, with intermediate demand to be determined in equilibrium as demand adjusts throughout the system. Thus, our demand shocks can propagate upstream as reductions in final demand in one sector translate to reductions

Table 22: Updated frequency of assessment period and adjusting price indices

Excess Failure Rates (COVID – non-COVID)

	Baseline	Half Year		Quarterly		Monthly		Weekly	
		Unadjusted	*Adjusted*	*Unadjusted*	*Adjusted*	*Unadjusted*	*Adjusted*	*Unadjusted*	*Adjusted*
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Czech Republic	2.56	3.07	4.25	3.23	4.28	3.66	5.29	3.67	5.32
Finland	4.18	6.22	8.46	6.62	9.56	7.19	12.55	7.18	13.05
France	5.27	6.53	5.45	7.05	4.60	7.21	5.35	7.29	5.58
Hungary	2.77	3.76	7.33	4.15	8.26	4.88	10.31	4.88	10.63
Italy	10.30	11.53	8.73	12.09	7.30	12.37	7.07	12.45	6.73
Poland	5.50	6.91	9.44	7.36	10.07	8.04	11.85	8.05	12.23
Portugal	4.02	5.35	4.15	5.51	2.98	5.97	2.58	5.97	2.28
Romania	2.37	3.40	7.06	3.68	8.15	4.46	12.05	4.45	12.65
Slovak Republic	3.02	3.51	2.84	3.70	2.60	4.00	2.79	4.00	2.75
Slovenia	2.98	4.65	4.71	5.22	4.81	6.29	6.04	6.30	5.98
Spain	3.75	4.77	4.03	4.94	3.73	5.30	3.89	5.31	3.87
All	6.01	7.22	6.40	7.65	5.77	7.97	6.39	8.02	6.43

Notes: Reports the excess failure rates (COVID - non-COVID) under the following scenarios that differ in the assessment period and on whether we redistribute demand as firms fail: Baseline, which reflects an annual assessment period (col. 1); bi-annual (cols. 2 and 3), quarterly (cols. 4 and 5), monthly, (cols. 6 and 7), and weekly (cols. 8 and 9). Failure rates are first calculated at the 1-digit NACE level and aggregated to the country level using 2018 sector GVA as weights. Failure rates are aggregated across countries using GVA as weights.

in intermediates and lower demand in other sectors. In order for supply shocks to propagate throughout the network, we allow firms to adjust their prices in response to shocks, and these price increases affect downstream customers through their intermediate usage. Full derivations and implementation details are in Appendix G.1.

In line with the literature, we focus on the case where final demand has unit elasticity ($\eta = 1$) but where inputs are complementary. We use a single elasticity σ that governs the choice between labor and materials, as well as between individual varieties of inputs. The production function is:

$$y_{is} = z_{is}\left(\alpha_s k_{is}^{\frac{\sigma-1}{\sigma}} + \beta_s (A_s L_{is})^{\frac{\sigma-1}{\sigma}} + \gamma_s \sum_k \vartheta_{sk} \int_0^{\mathcal{N}_k} x_{is,lk}^{\frac{\sigma-1}{\sigma}} dl\right)^{\frac{\sigma}{\sigma-1}}$$

which is an extension of the Cobb-Douglas production function in our paper. Allowing for $\sigma < 1$ means we can capture meaningfully the complimentarity between different intermediate inputs and how bottlenecks can form when one input becomes scarce.

Table 23 summarizes the main results from this exercise at the country level. Column (1) reports the baseline estimates with no I-O linkages, fixed prices and $\sigma = 1$. Column (2) reports the effect of allowing prices to be flexible and is a repeat of Column (2) in Table 20. Columns (3) and (4) report what happens in the flexible price case (still without I-O linkages) when the elasticity of substitution between materials and labor is varies, while material prices remain fixed. Once firms are in lockdown and become labor supply constrained, this elasticity will govern how simple it is to substitute labor with materials. Columns (5)-(7) repeat columns (2)-(4) with I-O linkages.

Table 23: I-O linkages at the country level

Excess Failure Rates (COVID – non-COVID)

		Flexible Prices			+ I-O linkages		
	Baseline	$(\sigma = 1)$	$(\sigma = 0.2)$	$(\sigma = 4)$	$(\sigma = 1)$	$(\sigma = 0.2)$	$(\sigma = 4)$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Czech Republic	2.56	4.94	8.40	-0.57	5.89	9.22	4.05
Finland	4.18	8.54	15.51	1.31	6.18	12.14	3.18
France	5.27	14.97	24.90	7.29	13.92	20.96	8.60
Hungary	2.77	5.81	10.77	-2.55	5.25	7.22	3.59
Italy	10.30	18.69	28.28	7.83	16.79	25.47	8.90
Poland	5.50	9.65	14.74	-6.55	8.66	14.56	5.76
Portugal	4.02	11.03	19.50	4.78	7.39	14.56	3.33
Romania	2.37	4.29	4.51	-1.31	4.64	7.54	3.08
Slovak Republic	3.02	6.39	10.48	1.31	6.39	9.98	4.26
Slovenia	2.98	9.30	16.99	4.28	7.23	13.19	4.03
Spain	3.75	9.31	18.25	4.22	7.20	15.51	2.22
All	6.01	13.37	21.93	4.95	11.92	19.13	6.59

Notes: Reports the excess failure rate (COVID - non-COVID) under scenarios with flexible prices with different input elasticities σ — (1) shows our baseline with fixed prices, (2)-(4) scenarios with flexible prices and (5)-(7) with the addition of input-output linkages. When aggregating sectoral shocks we weight by each sector's value added. Sector excess failure rates (Δ) are first calculated at the 1-digit NACE level for each country, and then aggregated across sectors using (country x sector) GVA as weights. The last row is the sector GVA weighted average.

Broadly speaking, I-O linkages have small effects at the country level, with overall excess failure rates broadly similar to their flexible price counterparts, when elasticities of substitution are similar. However, these country level results mask considerable heterogeneity at the sectoral level. As Table 24 shows, differences in excess failure rates across scenerios are considerably higher. For example, Transport & Storage has considerably higher excess failure rates once I-O linkages are accounted for, while Mining has considerably lower excess failure rates—especially when $\sigma = 0.2$. Surprisingly, we don't find much affect of I-O linkages excess failure rates in Manufacturing.

G.1 Details to Implement I-O linkages

In this section we enrich the supply side of the model. We allow firms to adjust prices in response to shocks, and connect these price changes to intermediate prices of other firms, using I-O data from WIDO.

Demand: Firm sales is now the combination of two terms: sales to households $p_{ij}d_{ij}$ and sales to other firms $p_{ij} \sum_j \sum_k x_{ij,lk}$, where $x_{ij,lk}$ denotes sales by firm i in sector j selling to firm l

Table 24: I-O linkages at the sectoral level

Excess Failure Rates (COVID – non-COVID)

		Flexible Prices			+ I-O linkages		
	Baseline	$(\sigma = 1)$	$(\sigma = 0.2)$	$(\sigma = 4)$	$(\sigma = 1)$	$(\sigma = 0.2)$	$(\sigma = 4)$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Agriculture	0.98	0.00	0.47	-3.10	1.74	-0.93	1.24
Mining	4.70	2.32	0.03	-1.01	1.18	-5.38	1.62
Manufacturing	1.92	1.20	3.59	0.88	3.14	1.70	3.14
Electric, Gas & Air Con	0.12	0.90	2.94	-4.14	3.78	1.88	2.30
Water & Waste	-0.47	2.17	5.19	2.37	6.37	7.55	3.88
Construction	0.10	3.78	8.82	-1.73	5.65	9.02	3.48
Wholesale & Retail	8.87	19.39	35.48	-2.85	12.48	24.92	4.98
Transport & Storage	1.56	6.63	12.78	1.71	15.29	14.94	11.38
Accom. & Food Service	13.31	26.96	49.04	11.24	20.02	38.56	11.03
Info. & Comms	3.68	16.39	29.55	5.73	15.69	26.19	9.83
Real Estate	5.97	15.35	24.24	8.27	12.66	25.62	-0.42
Prof., Sci., & Technical	6.79	17.58	26.43	9.56	16.39	27.19	8.88
Administration	11.02	19.55	28.39	10.10	25.93	36.38	18.03
Education	18.74	37.07	48.58	24.43	28.82	36.75	26.83
Health & Social Work	2.51	5.82	10.54	3.02	5.01	8.15	3.06
Arts, Ent., & Recreation	18.65	37.89	50.71	23.35	16.70	34.51	9.43
Other Services	14.22	29.58	40.35	18.92	12.24	35.07	4.00

Notes: Reports the excess failure rate (COVID - non-COVID) under scenarios with flexible prices with different input elasticities σ — (1) shows our baseline with fixed prices, (2)-(4) scenarios with flexible prices and (5)-(7) with the addition of input-output linkages. When aggregating sectoral shocks we weight by each sector's value added. Sector excess failure rates (Δ) are first calculated at the 1-digit NACE level for each country, and then aggregated across countries using (country x sector) GVA as weights.

in sector k:

$$p_{ij}y_{ij} = p_{is}d_{ij} + p_{ij}\sum_j \sum_k x_{ij,lk} \tag{40}$$

Final demand is given by the Equations (3), (4) and (5) in the main text, where we specialize to the case where $\eta = 1$. This gives us a new Equation 7:

$$\hat{p}_{ij}\hat{d}_{ij} = \underbrace{\frac{\hat{\xi}_j}{\sum_\sigma \hat{\xi}_\sigma / S}}_{\equiv e^{\tilde{\xi}_j}} \widehat{PD} \tag{41}$$

Intermediate demand $x_{ij,lk}$ will come from the demand for intermediates from the supply block of the model:

Supply: Firm level production is given by the following production function:

$$y_{is} = z_{is}\left(\alpha_j k_{ij}^{\frac{\sigma-1}{\sigma}} + \beta_j n_{ij}^{\frac{\sigma-1}{\sigma}} + \gamma_j \sum_k \int_0^{\mathcal{N}_k} \vartheta_{j,k} x_{ij,lk}^{\frac{\sigma-1}{\sigma}} dl\right)^{\frac{\sigma}{\sigma-1}} \tag{42}$$

When sectoral wages are w_j and intermediate prices are p_{lk}, firm i has the following input demands:

$$n_{ij} = \left(\frac{\beta_j p_{ij}}{w_j}\right)^{\sigma} (A_j z_{ij})^{\sigma-1} y_{ij} \tag{43}$$

$$x_{ij,lk} = \left(\frac{\gamma_j \vartheta_{jk} p_{ij}}{p_{lk}}\right)^{\sigma} z_{ij}^{\sigma-1} y_{ij} \tag{44}$$

In COVID, we impose that $n_j \leq \hat{x}_j \bar{N}_j$ and that wages are fixed $w_j = \bar{w}_j$. To see if this constraint binds we solve for the flexible wage w_j^{flex} that clears the sectoral labor market for a given level of output prices and production. Equating labor supply ($x_j \bar{N}_j$) to labor demand, $\int_0^{\mathcal{N}_j} n_{ij} di = \int_0^{\mathcal{N}_j} \left(\frac{\beta_j p_{ij}}{w_j^{flex}}\right)^{\sigma} (A_j z_{ij})^{\sigma-1} y_{ij} di$. With common multiplicative shocks to firms within a sector and applying hat algebra gives us the following expression for the flexible wage change in COVID:

$$\hat{w}_j^{flex} = \hat{p}_j \left(\frac{\hat{y}_j}{\hat{x}_j}\right)^{\frac{1}{\sigma}} \hat{A}_j^{\frac{\sigma-1}{\sigma}} \tag{45}$$

The change in labor that an unrestricted firm would choose at a wage of $\bar{w}_j$ is given by:

$$\hat{n}_j^{unrestricted} = \hat{p}_j^{\sigma} \hat{A}_j^{\sigma-1} \hat{y}_j \tag{46}$$

Putting these two equations together with the labor constraint gives us:

$$\hat{n}_j = \min\left\{\hat{x}_j, \hat{x}_j \left(\hat{w}_j^{flex}\right)^{\sigma}\right\} \tag{47}$$

We also define a shadow wage $\hat{w}_j^s \equiv max\left\{1, \hat{w}_j^{flex}\right\}$. Using this shadow wage we can substitute intermediate demand Eq. (44) and Eq. (43) back into the production function Eq. (42) to obtain the following log-linearized expression:

$$\left(s_{nj} + s_{mj}\right) \check{p}_j = \frac{1}{\sigma}\left(1 - s_{nj} - s_{mj}\right) \check{y}_j + s_{nj}(\check{w}_j^s - \check{A}_j) + \sum_k \Omega_{jk}^x \check{p}_k \tag{48}$$

where Ω^x_{jk} is the expenditure share firms in sector j spend on goods produced in sector k. We use the notation $\check{x} \equiv log(x'/x) = log(\hat{x})$ to denote the log change in x.

Log-linearizing Eq. (40) and replacing $\hat{d}_j$ from Eq. (41) and $\hat{x}_{lk,ij}$ from Eq. (44) we obtain:

$$\lambda_j \check{y}_j = \omega_j \left(\check{\tilde{\xi}}_j + P\check{D}\right) - (\sigma\lambda_j + (1-\sigma)\omega_j)\check{p}_j + \sum_k (\sigma\check{p}_k + \check{y}_k)\Omega^x_{kj}\lambda_k \tag{49}$$

where $\lambda_j = \int_0^{\mathcal{N}_j} p_{ij}y_{ij}di$ denotes the Domar weight for industry j (i.e. the ratio of gross sectoral output to total value added); ω_j denotes the expenditure share of final consumption on goods produced by j and we used the fact that goods market equilibrium Eq. (40) links the Domar weights λ_j to the final expenditure shares ω_j by $\lambda_j = \omega_j + \sum_k \Omega^x_{kj}\lambda_k$.

Finally we can evaluate firm profits as:

$$\pi'_{ij} = py_{ij}\left(e^{\check{p}_j}e^{\check{y}_j} - s_{ni}e^{\check{n}_j} - s_{mi}\sum_k \Omega^x_{kj}(e^{\check{p}_k}e^{\check{m}_{j,k}})\right) \tag{50}$$

This expression can be plugged into Equations 26 and 27 in the main paper to evaluate firm cash flow and then to evaluate firm failures.

Implementation: We obtain data from the World Input Output Database (WIDO) to construct a domestic I-O matrix that measures the fraction of intermediate inputs that each industry purchases from itself and all other industries.[64] We also obtain information on each industry's intermediate input share s_{mj} and share of total gross output from WIDO. Finally, we use country, sector-specific data on compensation of employees from OECD National Accounts to calibrate the labor share s_{nj}.

Additional Details on Solving the Model with I-O linkages We first rewrite Eqs. (48) and (49) in matrix form. Define the $J \times J$ matrix Ω whose (j,k) element is Ω^x_{jk}, and the $J \times 1$ vectors $\Omega^\ell \equiv \{s_{nj}\}$, $\Omega^x =\equiv \{s_{mj}\}$, $\hat{p}$, $\hat{y}$, $\hat{A}$, $\hat{\tilde{\xi}}$, $\hat{\tilde{w}}^s$, $\hat{\mathcal{N}}$, λ and ω with corresponding elements for each sector j. We can write the demand and supply blocs in matrix form as:

$$\begin{aligned} \left(\Omega^\ell + \Omega^x\right)\circ\check{p} &= \frac{1}{\sigma}(1-\Omega^\ell-\Omega^x)\circ\check{y} + \Omega^\ell\circ(\check{\tilde{w}}^s - \check{A}) + \Omega\check{p} && \text{(51a)} \\ \left(I-\Omega^T\right)(\lambda\circ\check{y}) &= \omega\circ\left(\check{\tilde{\xi}} + P\check{D}\right) - \left[\sigma\lambda + (1-\sigma)\,\omega\right]\circ\check{p} + \sigma\Omega^T(\lambda\circ\check{p}), && \text{(51b)} \end{aligned}$$

[64] WIDO aggregates 2-digit ISIC codes into 56 sectors. Our analysis excludes the Financial and Insurance Activities (1-digit sector K), Public Administration (O), Activities of Households as Employers (T), and Activities of Extraterrestrial Organizations (U) sectors, which are included in WIDO I-O tables. We redistribute intermediate input purchases of these excluded sectors to all remaining sectors based on the intermediate input shares of each excluded sector.

where the notation $x \circ y$ denotes the Hadamard product of vectors x and y (i.e. element by element multiplication) and Ω^T is the matrix transpose of Ω.

Eq. (51) constitutes a linear system of $2J$ equations with $2J$ unknown ($\hat{p}$ and $\hat{y}$), given the shocks $\tilde{\xi}$, $\hat{A}$ and $\widehat{PD}$, wages $\hat{w}^s$.

Next, note that for generic vectors x and y, we can write $x \circ y = \text{Diag}_x y$ where Diag_x is a diagonal matrix with vector x inserted on the diagonal. It follows that we can solve the linear system as follows:

$$\check{p} = \Psi^{-1}\left(\text{Diag}_{1-\Omega^\ell-\Omega^x}\text{Diag}_\lambda^{-1}\left(I-\Omega^T\right)^{-1}\text{Diag}_\omega\left(\tilde{\xi}+P\check{D}\right)+\sigma\text{Diag}_{\Omega^\ell}(\check{w}^s-\check{A})\right) \tag{52a}$$

$$\begin{aligned}\hat{y} = {} & \text{Diag}_\lambda^{-1}\left(I-\Omega^T\right)^{-1}\text{Diag}_\omega\left(\tilde{\xi}+P\check{D}\right) \\ & -\left(\sigma+(1-\sigma)\,\text{Diag}_\lambda^{-1}\left(I-\Omega^T\right)^{-1}\text{Diag}_\omega\right)\check{p},\end{aligned} \tag{52b}$$

where

$$\Psi = \sigma\left(I-\Omega\right)+(1-\sigma)\,\text{Diag}_{1-\Omega^\ell-\Omega^m}\text{Diag}_\lambda^{-1}\left(I-\Omega^T\right)^{-1}\text{Diag}_\omega, \tag{53}$$

and Ψ^{-1} denotes the matrix inverse of Ψ.

In vector form, the shadow wage satisfies (from Eq. (45)):

$$\sigma\check{w}^s = \max\left\langle 0, -\check{x}+\sigma\check{p}-(1-\sigma)\,\check{A}+\check{y}\right\rangle, \tag{54}$$

With these equations solved as a system, we can then evaluate the change in profits at the firm level Eq. (50) and then cashflow and then assess if the firm fails.

H Shock Decompositions of COVID

Table 8 in Section 5.3.2 reports excess failure rates, under our baseline set of assumption, for a set of shock combinations. In this section, we report excess failure rates for the full set of possible shock combinations. We do so by adding each shock separately to all other possible combinations of remaining shocks. Table 25 starts by only introducing the aggregate demand shock ($\widehat{PD}$) (col. 1). Then, the aggregate demand shock is combined individually with the labor supply constraint ($\hat{x}_s$), the sector-specific demand shock ($\tilde{\xi}_s$) and the sector-specific productivity shock in columns (2) through (4). Columns (5) to (7) combine the aggregate demand shock with different sets of *two* of the remaining shocks, and column (8) shows reports our baseline excess failure rates, where all shocks are active.

Tables 26, 27, and 28 repeat the same approach with sectoral supply ($\hat{x}_s$), sector-specific demand ($\hat{\xi}^{\eta}_s$) and sectoral productivity ($\hat{A}_s$) shocks, respectively.

Table 25: Excess Failure Rate (Δ): $\widehat{PD}$ Combinations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$\widehat{PD}$	$\widehat{PD}, \hat{x}_s$	$(\widehat{PD}\tilde{\xi}^{\eta}_s)$	$\widehat{PD}, \hat{A}_s$	$(\widehat{PD}\tilde{\xi}^{\eta}_s), \hat{x}_s$	$\widehat{PD}, \hat{x}_s, \hat{A}_s$	$(\widehat{PD}\tilde{\xi}^{\eta}_s), \hat{A}_s$	Baseline
Agriculture	0.73	1.26	0.38	0.61	0.97	1.18	0.37	0.98
Mining	0.05	4.12	0.41	-2.82	4.84	1.69	0.20	5.13
Manufacturing	1.04	2.13	0.75	0.83	1.97	2.06	0.58	1.92
Electric, Gas & Air Con	1.12	1.12	0.07	1.10	0.07	1.10	0.12	0.12
Water & Waste	3.60	3.60	0.49	2.12	0.49	2.12	-0.47	-0.47
Construction	1.81	1.85	-0.33	2.06	-0.34	2.12	0.10	0.10
Wholesale & Retail	2.18	2.86	8.76	2.65	8.56	3.38	9.07	8.87
Transport & Storage	7.23	7.24	1.21	7.53	1.22	7.53	1.55	1.56
Accom. & Food Service	0.09	10.27	7.85	1.42	11.79	10.38	8.40	13.31
Info. & Comms	1.77	1.92	3.15	2.37	3.15	2.69	3.68	3.68
Real Estate	1.60	0.97	6.04	1.65	6.03	0.97	6.02	5.97
Prof., Sci., & Technical	3.40	3.14	6.80	3.62	6.71	3.33	6.87	6.79
Administration	4.28	4.46	9.35	5.94	9.35	6.03	11.02	11.02
Education	2.35	12.73	19.01	3.39	19.01	15.13	19.49	19.49
Health & Social Work	2.11	3.48	2.50	2.50	2.50	3.60	2.59	2.59
Arts, Ent., & Recreation	1.88	10.60	18.58	3.09	18.82	11.89	19.19	19.37
Other Services	0.07	7.35	14.56	0.72	14.87	7.24	14.54	14.88
Average	2.16	3.71	5.36	2.50	5.72	4.07	5.58	6.01

Notes: Reports the excess failure rate (COVID - non-COVID) under eight scenarios—(1) aggregate demand shock only ($\widehat{PD}$); (2) aggregate demand and sectoral supply shocks ($\widehat{PD}, \hat{x}_s$); (3) aggregate demand and sector-specific demand shocks (total demand shock, $\widehat{PD}\tilde{\xi}^{\eta}_s$); (4) aggregate demand and sectoral productivity shocks ($\widehat{PD}, \hat{A}_s$); (5) total demand and supply shocks ($\widehat{PD}\tilde{\xi}^{\eta}_s, \hat{x}_s$); (6) aggregate demand, sectoral supply, and sectoral productivity shocks ($\widehat{PD}, \hat{x}_s, \hat{A}_s$); (7) total demand and sectoral productivity shocks ($\widehat{PD}\tilde{\xi}^{\eta}_s, \hat{A}_s$) and; (8) the baseline ($\widehat{PD}\tilde{\xi}^{\eta}_s, \hat{x}_s, \hat{A}_s$). Sector excess failure rates (Δ) are first calculated at the 1-digit NACE level for each country, and then aggregated across countries using (country x sector) GVA as weights. The last row is the sector GVA weighted average. 1-digit sectors where the majority of 4-digit sectors are classified as essential are highlighted in gray.

Table 26: Excess Failure Rate (Δ): $\hat{x}_s$ Combinations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$\hat{x}_s$	$\hat{x}_s, \widehat{PD}$	$\hat{x}_s, \hat{\xi}^{\eta}_s$	$\hat{x}_s, \hat{A}_s$	$\hat{x}_s, (\widehat{PD\tilde{\xi}^{\eta}_s})$	$\hat{x}_s, \widehat{PD}, \hat{A}_s$	$\hat{x}_s, \hat{\xi}^{\eta}_s, \hat{A}_s$	Baseline
Agriculture	0.66	1.26	2.93	0.57	0.97	1.18	2.84	0.98
Mining	5.47	4.12	7.01	4.41	4.84	1.69	6.75	5.13
Manufacturing	1.74	2.13	3.78	1.67	1.97	2.06	3.81	1.92
Electric, Gas & Air Con	0.00	1.12	3.40	-0.03	0.07	1.10	3.49	0.12
Water & Waste	0.00	3.60	7.15	-0.93	0.49	2.12	6.50	-0.47
Construction	0.10	1.85	5.23	0.36	-0.34	2.12	5.61	0.10
Wholesale & Retail	1.41	2.86	7.38	1.94	8.56	3.38	7.67	8.87
Transport & Storage	0.02	7.24	9.52	0.23	1.22	7.53	9.88	1.56
Accom. & Food Service	9.58	10.27	9.43	9.68	11.79	10.38	10.77	13.31
Info. & Comms	1.02	1.92	3.30	2.47	3.15	2.69	3.95	3.68
Real Estate	0.27	0.97	2.17	0.29	6.03	0.97	2.19	5.97
Prof., Sci., & Technical	0.48	3.14	4.75	0.80	6.71	3.33	4.97	6.79
Administration	0.80	4.46	10.68	2.34	9.35	6.03	12.12	11.02
Education	14.25	12.73	14.06	14.94	19.01	15.13	14.38	19.49
Health & Social Work	1.47	3.48	3.32	1.56	2.50	3.60	3.41	2.59
Arts, Ent., & Recreation	10.76	10.60	12.80	11.67	18.82	11.89	13.54	19.37
Other Services	7.70	7.35	12.33	7.56	14.87	7.24	12.34	14.88
Average	2.24	3.71	5.89	2.55	5.72	4.07	6.18	6.01

Notes: Reports the excess failure rate (COVID - non-COVID) under eight scenarios—(1) sectoral supply shocks only ($\hat{x}_s$); (2) sectoral supply and aggregate demand shocks ($\hat{x}_s, \widehat{PD}$); (3) sectoral supply and sector-specific demand shocks ($\hat{x}_s, \hat{\xi}^{\eta}_s$); (4) sectoral supply and sectoral productivity shocks ($\hat{x}_s, \hat{A}_s$); (5) sectoral supply and total demand shocks ($\hat{x}_s, \widehat{PD\tilde{\xi}^{\eta}_s}$); (6) sectoral supply, aggregate demand and sectoral productivity shocks ($\hat{x}_s, \widehat{PD}, \hat{A}_s$); (7) sectoral supply, sector-specific demand and sectoral productivity shocks ($\hat{x}_s, \hat{\xi}^{\eta}_s, \hat{A}_s$) and; (8) the baseline ($\hat{x}_s, \widehat{PD\tilde{\xi}^{\eta}_s}, \hat{A}_s$). Sector excess failure rates (Δ) are first calculated at the 1-digit NACE level for each country, and then aggregated across countries using (country x sector) GVA as weights. The last row is the sector GVA weighted average. 1-digit sectors where the majority of 4-digit sectors are classified as essential are highlighted in gray.

Table 27: Excess Failure Rate (Δ): $\hat{\xi}_s^{\eta}$ Combinations

	(1) $\hat{\xi}_s^{\eta}$	(2) $(\widehat{PD\tilde{\xi}_s^{\eta}})$	(3) $\hat{\xi}_s^{\eta}, \hat{x}_s$	(4) $\hat{\xi}_s^{\eta}, \hat{A}_s$	(5) $(\widehat{PD\tilde{\xi}_s^{\eta}}), \hat{x}_s$	(6) $(\widehat{PD\tilde{\xi}_s^{\eta}}), \hat{A}_s$	(7) $\hat{\xi}_s^{\eta}, \hat{x}_s, \hat{A}_s$	(8) Baseline
Agriculture	2.30	0.38	2.93	2.46	0.97	0.37	2.84	0.98
Mining	0.82	0.41	7.01	0.98	4.84	0.20	6.75	5.13
Manufacturing	2.01	0.75	3.78	2.00	1.97	0.58	3.81	1.92
Electric, Gas & Air Con	3.40	0.07	3.40	3.49	0.07	0.12	3.49	0.12
Water & Waste	7.15	0.49	7.15	6.50	0.49	-0.47	6.50	-0.47
Construction	5.24	-0.33	5.23	5.61	-0.34	0.10	5.61	0.10
Wholesale & Retail	7.48	8.76	7.38	7.79	8.56	9.07	7.67	8.87
Transport & Storage	9.50	1.21	9.52	9.86	1.22	1.55	9.88	1.56
Accom. & Food Service	4.19	7.85	9.43	4.80	11.79	8.40	10.77	13.31
Info. & Comms	3.30	3.15	3.30	3.95	3.15	3.68	3.95	3.68
Real Estate	2.25	6.04	2.17	2.30	6.03	6.02	2.19	5.97
Prof., Sci., & Technical	4.72	6.80	4.75	4.82	6.71	6.87	4.97	6.79
Administration	10.61	9.35	10.68	12.04	9.35	11.02	12.12	11.02
Education	14.06	19.01	14.06	14.38	19.01	19.49	14.38	19.49
Health & Social Work	3.32	2.50	3.32	3.41	2.50	2.59	3.41	2.59
Arts, Ent., & Recreation	12.63	18.58	12.80	13.37	18.82	19.19	13.54	19.37
Other Services	12.08	14.56	12.33	11.92	14.87	14.54	12.34	14.88
Average	5.35	5.36	5.89	5.61	5.72	5.58	6.18	6.01

Notes: Reports the excess failure rate (COVID - non-COVID) under eight scenarios—(1) sector-specific demand shocks only ($\hat{\xi}_s^{\eta}$); (2) sector-specific demand and aggregate demand shocks (total demand shock, $\widehat{PD\tilde{\xi}_s^{\eta}}$); (3) sector-specific demand and sectoral supply shocks ($\hat{\xi}_s^{\eta}, \hat{x}_s$); (4) sector-specific demand and sectoral productivity shocks ($\hat{\xi}_s^{\eta}, \hat{A}_s$); (5) total demand and sectoral supply shocks ($\widehat{PD\tilde{\xi}_s^{\eta}}, \hat{x}_s$); (6) total demand and sectoral productivity shocks ($\widehat{PD\tilde{\xi}_s^{\eta}}, \hat{A}_s$); (7) sector-specific demand, sectoral supply and sectoral productivity shocks ($\hat{\xi}_s^{\eta}, \hat{x}_s, \hat{A}_s$) and; (8) the baseline ($\widehat{PD\tilde{\xi}_s^{\eta}}, \hat{x}_s, \hat{A}_s$). Sector excess failure rates (Δ) are first calculated at the 1-digit NACE level for each country, and then aggregated across countries using (country x sector) GVA as weights. The last row is the sector GVA weighted average. 1-digit sectors where the majority of 4-digit sectors are classified as essential are highlighted in gray.

Table 28: Excess Failure Rate (Δ): $\hat{A}_s$ Combinations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$\hat{A}_s$	$\hat{A}_s, \widehat{PD}$	$\hat{A}_s, \hat{x}_s$	$\hat{A}_s, \hat{\xi}^{\eta}_s$	$\hat{A}_s, \widehat{PD}, \hat{x}_s$	$\hat{A}_s, (\widehat{PD\xi^{\eta}_s})$	$\hat{A}_s, \hat{x}_s, \hat{\xi}^{\eta}_s$	Baseline
Agriculture	-0.10	0.61	0.57	2.46	1.18	0.37	2.84	0.98
Mining	-1.26	-2.82	4.41	0.98	1.69	0.20	6.75	5.13
Manufacturing	-0.19	0.83	1.67	2.00	2.06	0.58	3.81	1.92
Electric, Gas & Air Con	-0.03	1.10	-0.03	3.49	1.10	0.12	3.49	0.12
Water & Waste	-0.93	2.12	-0.93	6.50	2.12	-0.47	6.50	-0.47
Construction	0.25	2.06	0.36	5.61	2.12	0.10	5.61	0.10
Wholesale & Retail	0.44	2.65	1.94	7.79	3.38	9.07	7.67	8.87
Transport & Storage	0.20	7.53	0.23	9.86	7.53	1.55	9.88	1.56
Accom. & Food Service	1.61	1.42	9.68	4.80	10.38	8.40	10.77	13.31
Info. & Comms	1.02	2.37	2.47	3.95	2.69	3.68	3.95	3.68
Real Estate	0.23	1.65	0.29	2.30	0.97	6.02	2.19	5.97
Prof., Sci., & Technical	0.49	3.62	0.80	4.82	3.33	6.87	4.97	6.79
Administration	1.59	5.94	2.34	12.04	6.03	11.02	12.12	11.02
Education	1.55	3.39	14.94	14.38	15.13	19.49	14.38	19.49
Health & Social Work	0.49	2.50	1.56	3.41	3.60	2.59	3.41	2.59
Arts, Ent., & Recreation	1.68	3.09	11.67	13.37	11.89	19.19	13.54	19.37
Other Services	1.47	0.72	7.56	11.92	7.24	14.54	12.34	14.88
Average	0.47	2.50	2.55	5.61	4.07	5.58	6.18	6.01

Notes: Reports the excess failure rate (COVID - non-COVID) under eight scenarios—(1) sectoral productivity shocks only ($\hat{A}_s$); (2) sectoral productivity and aggregate demand shocks ($\hat{A}_s, \widehat{PD}$); (3) sectoral productivity and sector-specific demand shocks ($\hat{A}_s, \hat{\xi}^{\eta}_s$); (4) sectoral productivity and sectoral supply shocks ($\hat{A}_s, \hat{x}_s$); (5) sectoral productivity, aggregate demand and sectoral supply shocks ($\hat{A}_s, \widehat{PD}, \hat{x}_s$); (6) sectoral productivity and total demand shocks ($\hat{A}_s, \widehat{PD\xi^{\eta}_s}$); (7) sectoral productivity, sectoral supply and sector-specific demand shocks ($\hat{A}_s, \hat{x}_s, \hat{\xi}^{\eta}_s$) and; (8) the baseline ($\hat{A}_s, \hat{x}_s, \widehat{PD\xi^{\eta}_s}$). Sector excess failure rates (Δ) are first calculated at the 1-digit NACE level for each country, and then aggregated across countries using (country x sector) GVA as weights. The last row is the sector GVA weighted average. 1-digit sectors where the majority of 4-digit sectors are classified as essential are highlighted in gray.

I Ex-post Analysis – Eurostat data flags

Table 29 reports the data flags associated with 2019 and 2020 Eurostat failure rates. It is worth noting that the majority of 2019 failure rates are still provisional, and the majority of 2020 failure rates are estimated. This is the case because it takes roughly three calendar years for the actual (close to final) statistics to be released.

Table 29: Eurostat Failure Rate Data Flags

	2019	2020
Czech Republic	P	E
Finland		
France		P
Hungary	P	B & E
Italy	P	E
Poland	P	P
Portugal	P	E
Romania	P	E
Slovak Republic	P	E
Slovenia		P
Spain		E

Notes: Reports the data flags associated with employer firm failure rates reported by Eurostat for 2019 and 2020. P = provisional, E = estimated and B = break in the time series.

J Policy Tables with multiple measures of Firm Value

We repeat Tables 11 and 13 with three measures of firm value:

1. Firm net worth defined as the book value of assets less liabilities. This version of firm value is shown in the original tables.

2. Firm Liquidation Value defined as the book value of tangible fixed assets.

3. Firm Going Concern Value defined as the book value of assets plus sales for the year.

K Calibrating policy to match amounts and take-up data

We consider three types of fiscal support policies are calibrated to match real-world aggregate policy costs. The policies we consider are (a) tax waivers where firms do not need to pay

Table 30: The Impact and Costs of Various Policy Scenarios

	(1) Firms Saved (% Firms)	(2) Jobs Saved (% Employed)	(3) Wages Saved (% GVA)	(4) Net Worth Prserved (% GVA)	(5) Liquidation Value (% GVA)	(6) Going Concern Value (% GVA)	(7) Funds Disbursed* (% GVA)
Benchmark Policy	7.29	3.10	1.44	15.49	6.30	23.25	0.77
Financial Expenses Waived	1.67	0.66	0.33	7.43	5.71	10.01	1.43
Tax Waiver	2.21	0.80	0.28	3.05	1.94	4.27	1.61
Rent Waiver	3.97	2.22	0.97	5.24	1.73	9.22	3.05
Cash Grant	4.74	2.63	1.14	4.96	2.12	7.70	2.63
Pandemic Loans	7.85	4.02	1.80	10.44	3.88	16.59	6.43

Notes: Because Orbis does not cover the universe of firms, we calculate aggregate costs by scaling the total costs in Orbis by the inverse of the coverage ratio of Orbis (based on 2018 value added for policy costs, total remuneration for wages saved, and employment at the 1-digit NACE level). The numbers presented here are GDP-weighted averages across countries.

* Unlike the other policies, the funds disbursed under the pandemic loan policy do not equal the fiscal cost, which depends on the rate of repayment and the distribution of losses between the government and the banking sector.

Table 31: Wages, Jobs and Loans Saved by Firm Type

	Jobs Saved (% Emp)	Wages Saved (% GVA)	Net worth Preserved (% GVA)	Liquidation Preserved (% GVA)	Going Concern Preserved (% GVA)	Policy Cost* (% GVA)
	Firms Bankrupt Regardless of COVID (Weak Firms)					
Benchmark Policy	0.00	0.00	0.00	0.00	0.00	0.00
Cash Grant	1.10	0.48	1.44	0.60	2.32	0.19
Pandemic Loans	1.80	0.82	3.53	1.35	5.78	0.45
	Firms Bankrupt Only in COVID Scenario (Viable Firms)					
Benchmark policy	3.11	1.44	15.49	6.30	23.25	0.77
Cash Grant	1.53	0.66	3.52	1.52	5.37	0.19
Pandemic Loans	2.23	0.98	6.90	2.53	10.81	0.53

Notes: Because Orbis does not cover the universe of firms, we calculate aggregate costs by scaling the total costs in Orbis by the inverse of the coverage ratio of Orbis (based on 2018 value added at the 1-digit NACE level). The numbers presented here are GDP-weighted averages. * Unlike the other policies, the funds disbursed under the pandemic loan policy do not equal the fiscal cost, which depends on the rate of repayment and the distribution of losses between the government and the banking sector.

a portion their 2020 tax bill due for 2020; (b) cash grants equal to a fraction of firms' pre-COVID labor costs and (c) government guaranteed loans that we refer to as 'pandemic loans'.[65] Starting with pandemic loans, we adopt a disbursement formula for firm i in country c broadly similar to that implemented by several Euro-area countries:

$$P_{i,c} = \theta_{c,loan} \max\{\text{Revenue}_{i,2018}, 2 \cdot \text{Labor costs}_{i,2018}\}. \tag{55}$$

In this formula, $\theta_{c,loan}$ is a parameter calibrated to match the overall amount disbursed under that policy in that country. We calibrate similarly the other two policies with parameters $\theta_{c,tax}$ and $\theta_{c,grant}$ that can vary based on the length of availability of the policy support and its generosity. We assume that all policy support is paid out in week 10 of 2020.[66]

[65] We exclude from our analysis policies – such as rent waivers or interest waivers – for which we lack estimates of their overall fiscal cost.

[66] We showed in a previous version of this paper that varying the timing of policy has very limited effect on overall firm failures. Moreover, all countries in our sample other than China and Korea first implemented

We use data from a variety of sources to calibrate the parameters $\{\theta_{c,tax}, \theta_{c,grant}, \theta_{c,loan}\}$ to both match the aggregate amounts of announced policy and adjust for less than full take-up of the various policies by firms. Specifically, we use OECD (2021) to check which countries used which policies. We then use data on policy costs from the European Systemic Risk Board (ESRB, 2021).

Table 32 shows the announced policy costs for each of the three policies for each country in our sample.[67] These numbers reported reflect the announced size of policies. Amounts disbursed may be lower due if governments imperfectly estimate the set of eligible firms, or firms neglect to apply for support. We were unable to find country level information on take-up by country. Instead, we use ESRB (2021)'s average take up rates for the sample of countries they cover and assume that take-up was the same in all countries in our sample, equal to these average numbers.

To map these aggregate costs to estimate each $\theta_{c,p}$, we first calculate and estimate the aggregate cost of policy p, $\text{Cost}_{c,p}$, under the assumption that $\theta_{c,p} = 1$ (equivalent to 1 year of policy support at 100% of the disbursement formula), adjusting for the fact that Orbis data only covers a subset of all firms in a country. Then we estimate $\theta_{c,p}$ by scaling up or down to the actual policy cost. Mathematically:[68]

lockdowns between weeks 8 and 12 of 2020 with a median of 10 weeks and announced policy support programs soon after.

[67] An asterisk indicates that the policy cost number was imputed as described above.

[68] The denominator is calculated as follows:

$$\text{Cost}_{c,p} = \sum_s \left(\frac{VA_s^{Orbis}}{VA_s} \right)^{-1} \sum_{i \in i(s)} Cost_{i,p}^{Orbis}$$

We scale each 1-digit NACE sector s by the inverse of the share of value added captured by Orbis ($\left(\frac{VA_s^{Orbis}}{VA_s}\right)^{-1}$) to ensure that our calculated policy costs are representative of the whole economy.

Table 32: Announced Policy Costs by Type and Country

Country	Source	Costs (% of GDP)			
		Tax Waiver	Pandemic Loans	Cash Grant	Total
Czech Republic	ESRB	1.24	14.87	1.70	17.81
Finland	ESRB	1.87	1.75	1.71	5.33
France	ESRB	2.14	12.37	1.64	16.15
Greece	ESRB	0.66	1.71	3.01	5.38
Hungary	ESRB	0.90	5.76	0.89	7.56
Italy	ESRB	1.29	21.59	2.19	25.08
Poland	ESRB	0.03	16.08	1.54*	17.64
Portugal	ESRB	0.30	6.35	0.97	7.63
Romania	ESRB	0.06	0.10	0.00	0.16
Slovak Republic	ESRB	0.49	4.34	1.77*	6.61
Slovenia	ESRB	2.19	4.58	1.77*	8.55
Spain	ESRB	0.92	11.78	1.51	14.21

Notes: * indicates if the policy was imputed from the average of its group (either Advanced or Emerging).

$$\theta_{c,p} = \frac{\text{Actual Cost}_{c,p}}{\text{Cost}_{c,p}}$$

The numerator is the actual cost (adjusted for take-up) and the denominator is the cost of providing 1 year of 100% policy support. Therefore if we calculate for a country that a policy would cost 2% of GDP if implemented at 100% for a year and in the data it cost 1.5% of GDP, then $\theta_{c,p} = \frac{1.5}{2} = 0.75$.

The scaling factor applies to each firm. This means that we are agnostic about potential positive selection into policy support by firms, or negative selection from turning down applicants.

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